Lightning Modeling Workshop • Albuquerque, NM • 1-3 April 2024

Models of Lightning Leaders and the Return Stroke

Caitano L. da Silva, J. G. Pantuso, J. Wemhoner,
D. P. Jensen, A. F. R. Leal, R. G. Sonnenfeld

LANGMUIR LABORATORY FOR ATMOSPHERIC RESEARCH

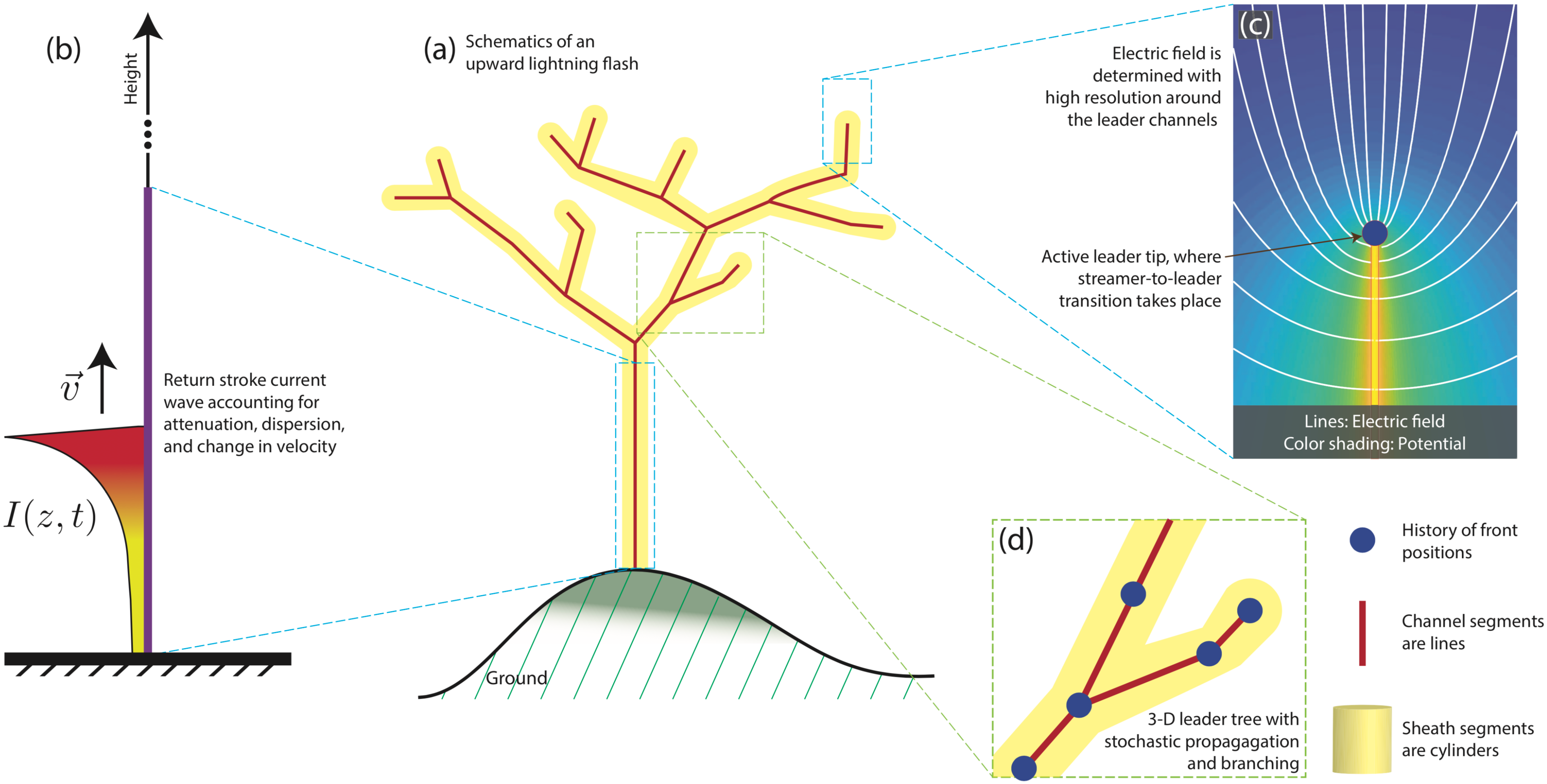
Department of Physics & Langmuir Lab
New Mexico Tech, Socorro, NM 87801
Email contact: caitano.dasilva@nmt.edu

NEW MEXICO TECH
SCIENCE • ENGINEERING • RESEARCH • UNIVERSITY

Outline

1. The Challenge
2. Leaders
3. Dart Leaders
4. Return Stroke
5. Plasma modeling
6. Summary

The Lightning Modeling Challenge



- Multiscale: in space (1 μm to 10s km) and time (>1 μs to ~1 s).
- Multiphysics: Electrodynamics, Plasma Physics, Chemistry, Meteorology.
- Multiregime: e.g. LTE (return stroke) versus non-LTE (streamer zones).
- Stochasticity.
- Validation.

New Mexico Tech's Efforts

CAREER: Self-consistent and Data-constrained Simulations of the Leader and Return Stroke Processes in Lightning Discharges (AGS-2046043)

- NSF/AGS/PDM
- Program officer:
  Dr. Chungu Lu
- Aug 2021–2026

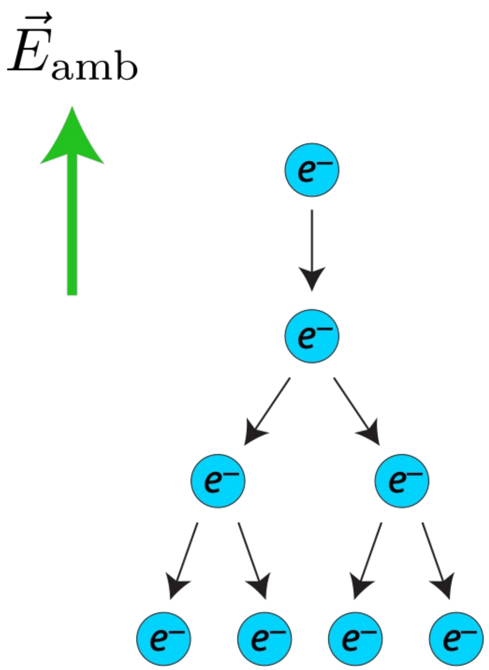
NEW MEXICO TECH
SCIENCE • ENGINEERING • RESEARCH UNIVERSITY

Background photo: Langmuir Lab   Credit: Dr. Harald Edens

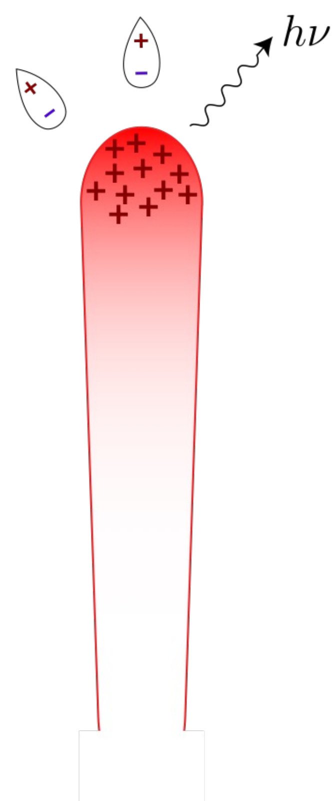
Leaders

Elements of Electrical Breakdown

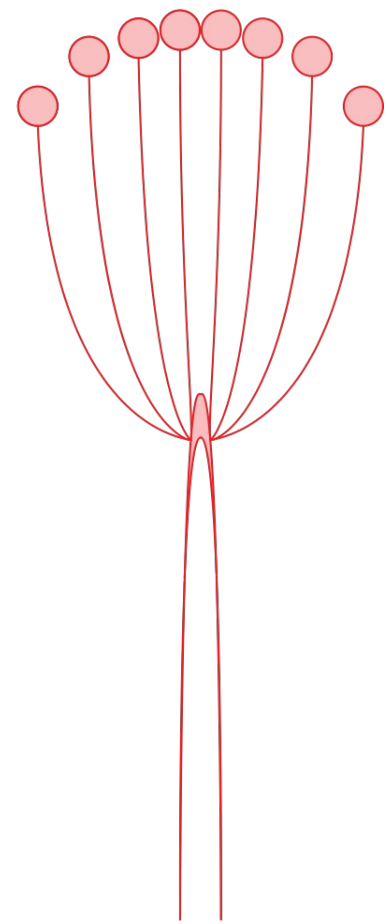
Avalanches



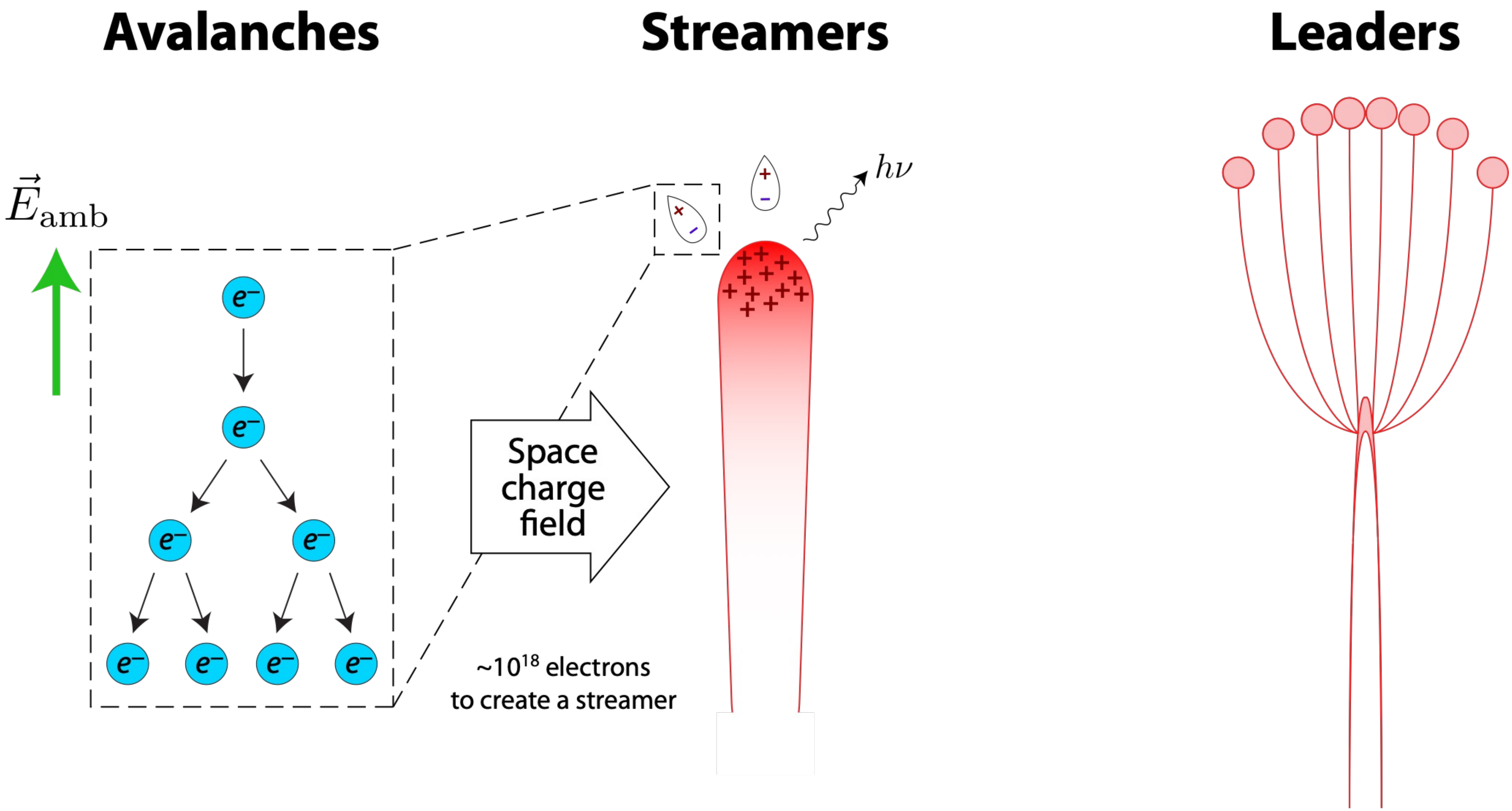
Streamers



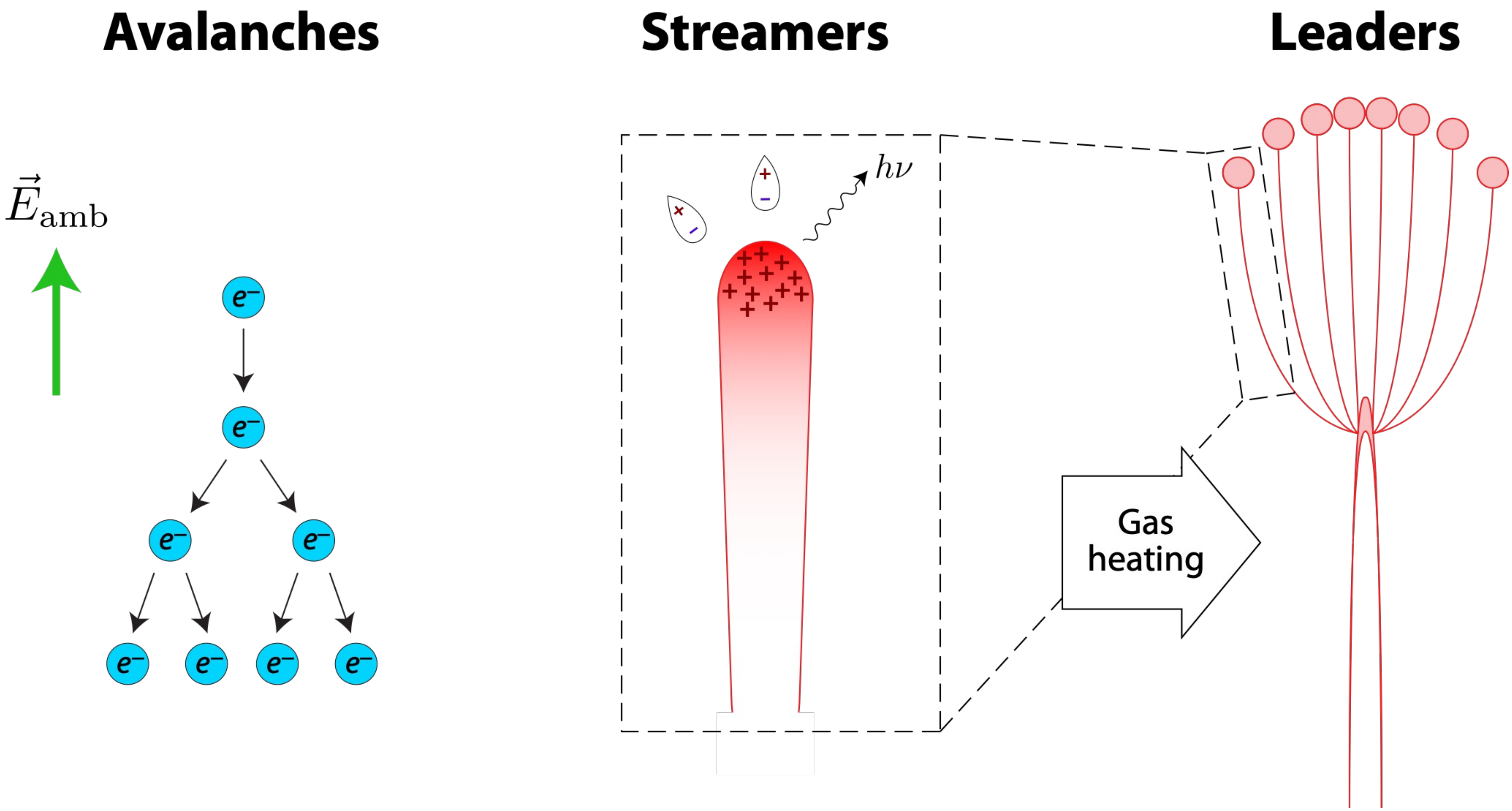
Leaders



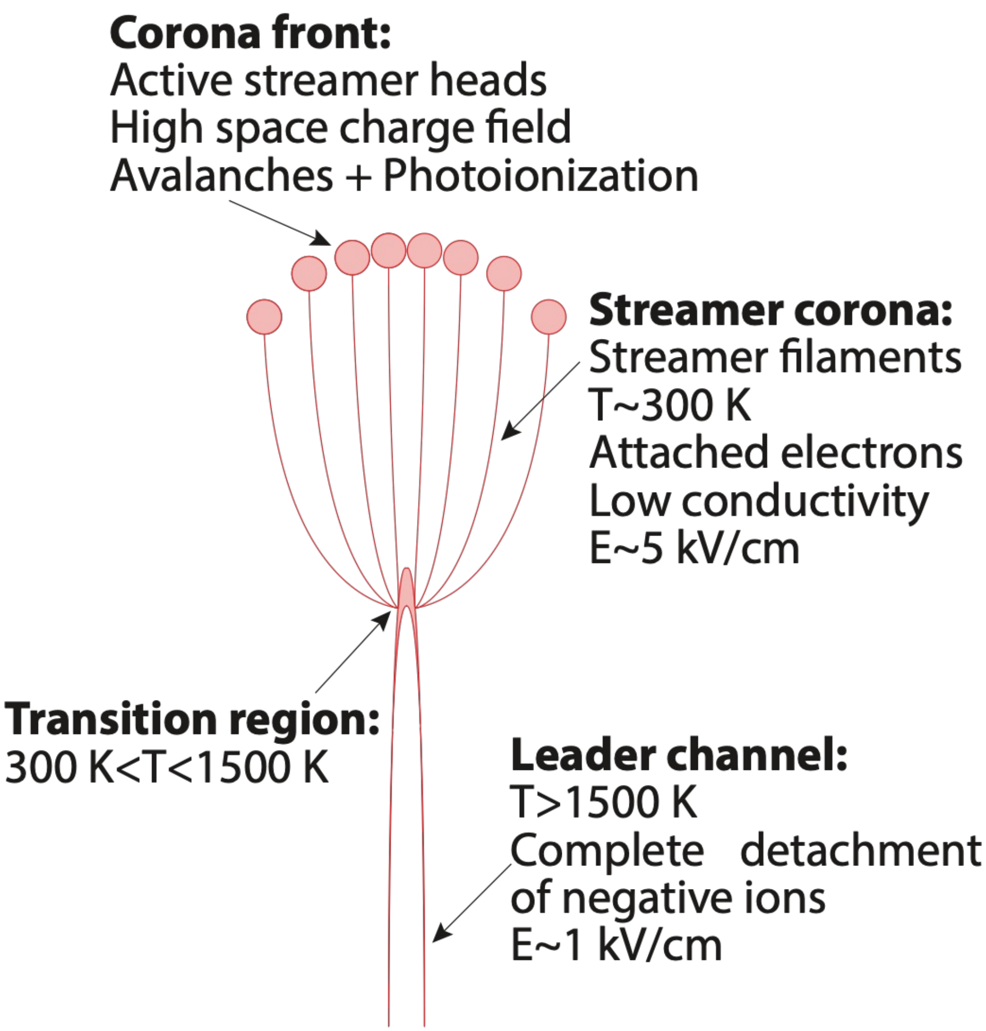
Avalanche-to-Streamer Transition



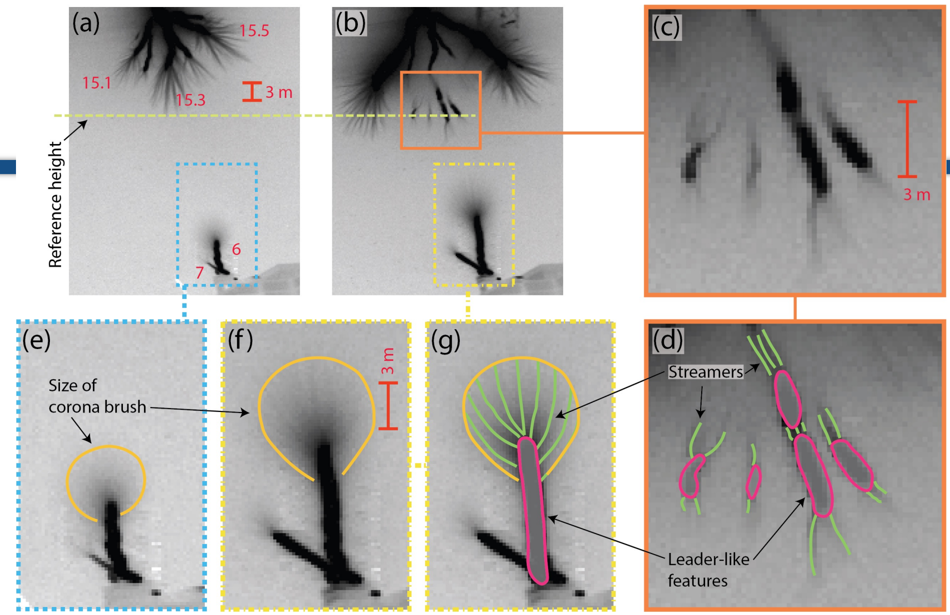
Streamer-to-Leader Transition



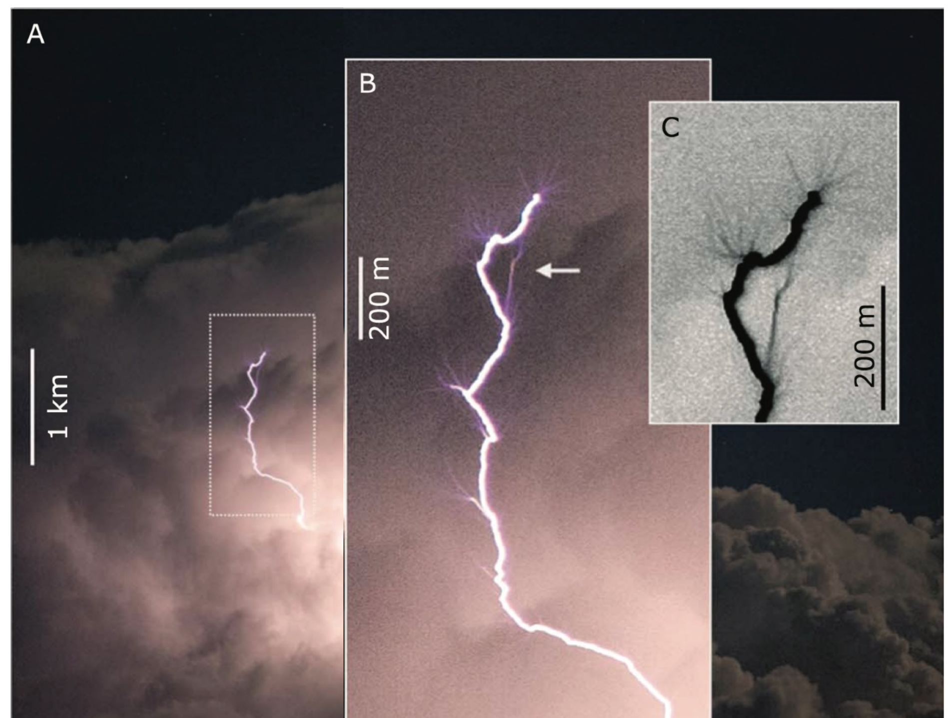
Leaders



Bondiou and Gallimberti [JPD, 1994]



Saba et al. [GRL, 2022]



Edens et al. [JGR, 2014]

Towards Physics--Based

Leader Modeling (positive, unbranched)

Electrodynamics: Quasi-electrostatic approximation

Charge: continuity equation

$$\frac{\partial q_i}{\partial t} = I_j - I_{j+1}$$

Current: Ohm's Law

$$I = \sigma \pi r_I^2 E$$

Electric field

$$E_j = -\frac{V_i - V_{i-1}}{\Delta s}$$

Electric potential

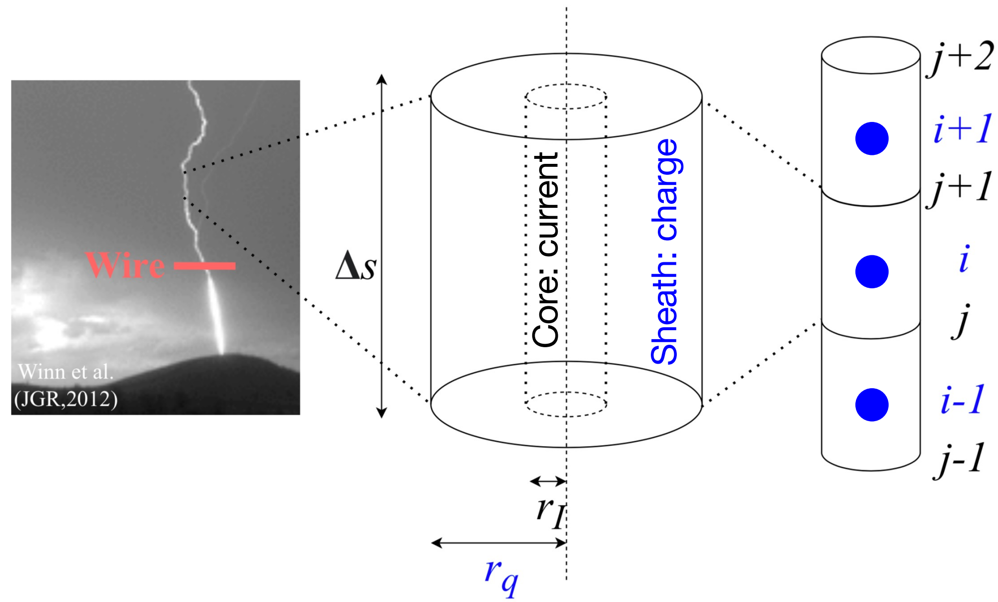
$$V_i = V_{\mathrm{amb},i} + \sum_m K_{im} q_m$$

Plasma Phys.

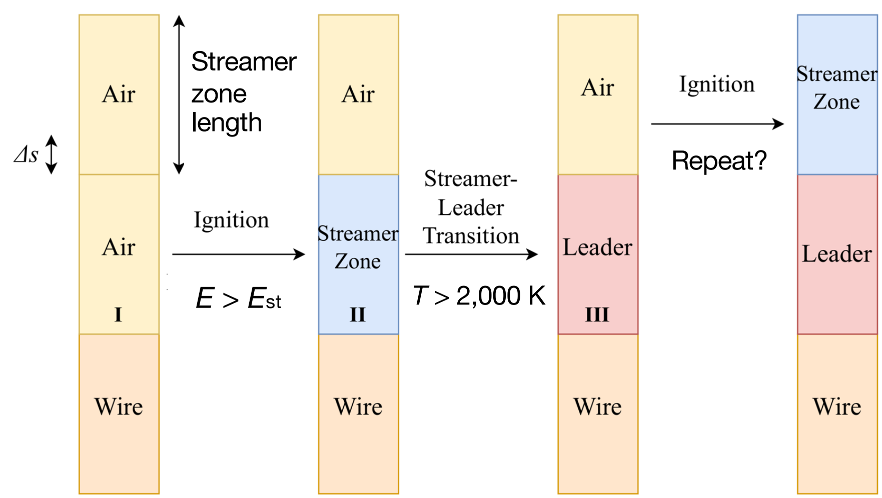
Conductivity

σ Plasma model explained later in this talk

✦ Computational discretization



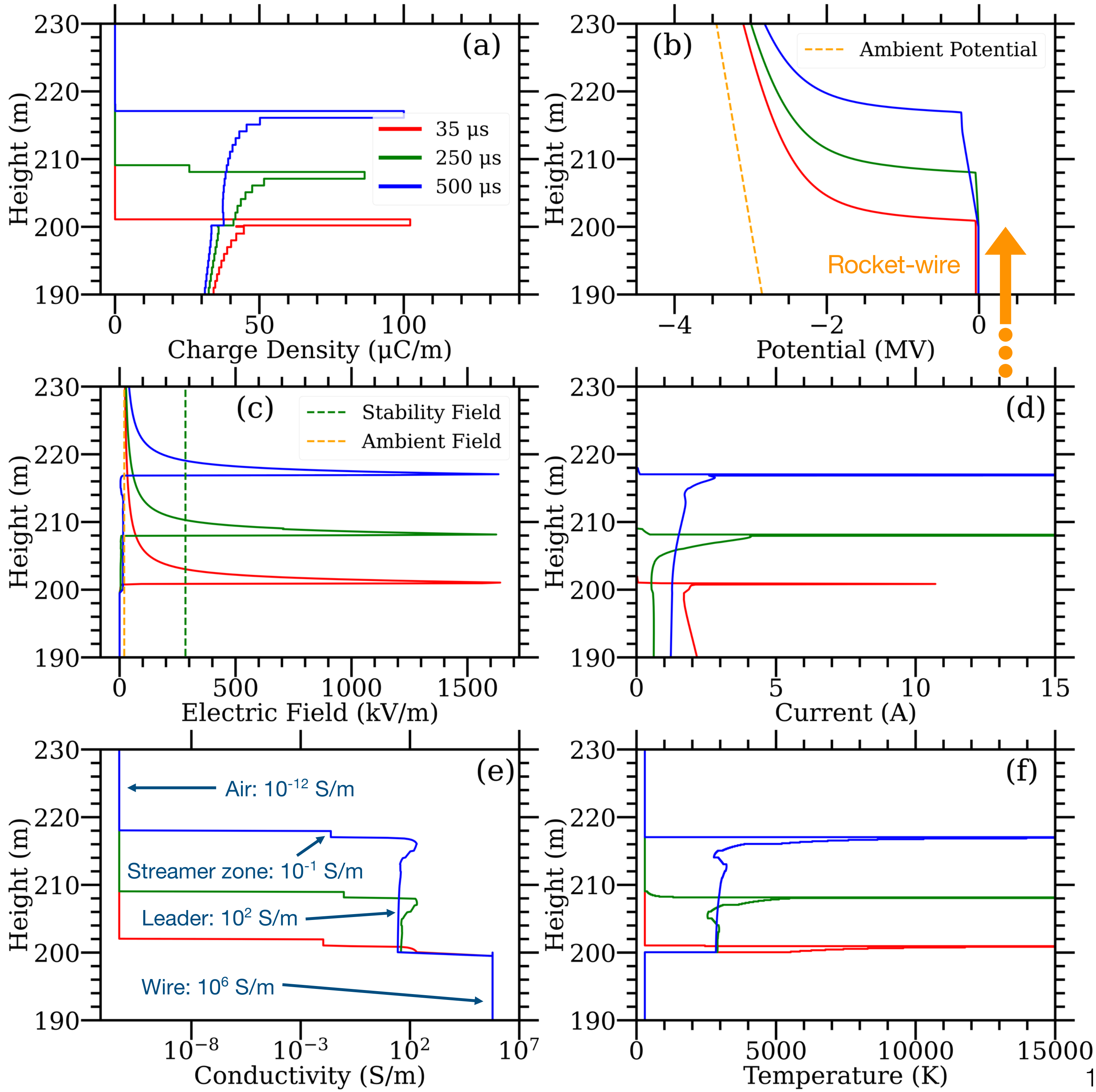
✦ Propagation procedure



Physics-Inspired, Self-Consistent Model of Upward Positive Leaders: **Results**

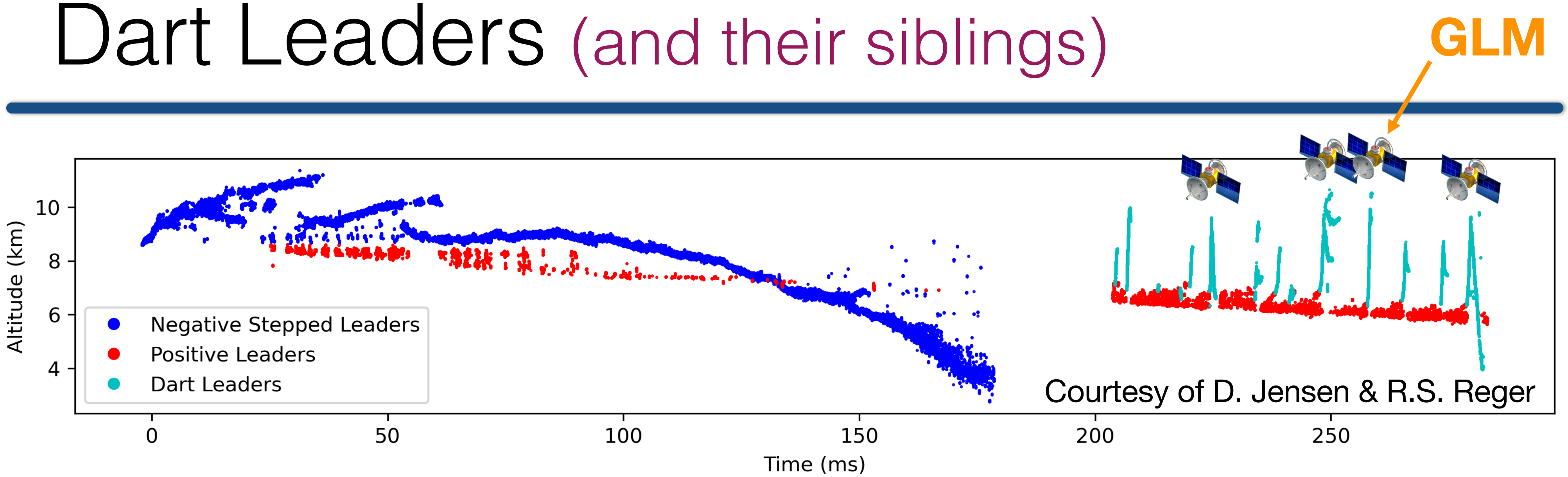
- Rocket-wire system height = 200 m
- Ambient field underneath storm = 15 kV/m
- Predicted speed = 5×10^4 m/s

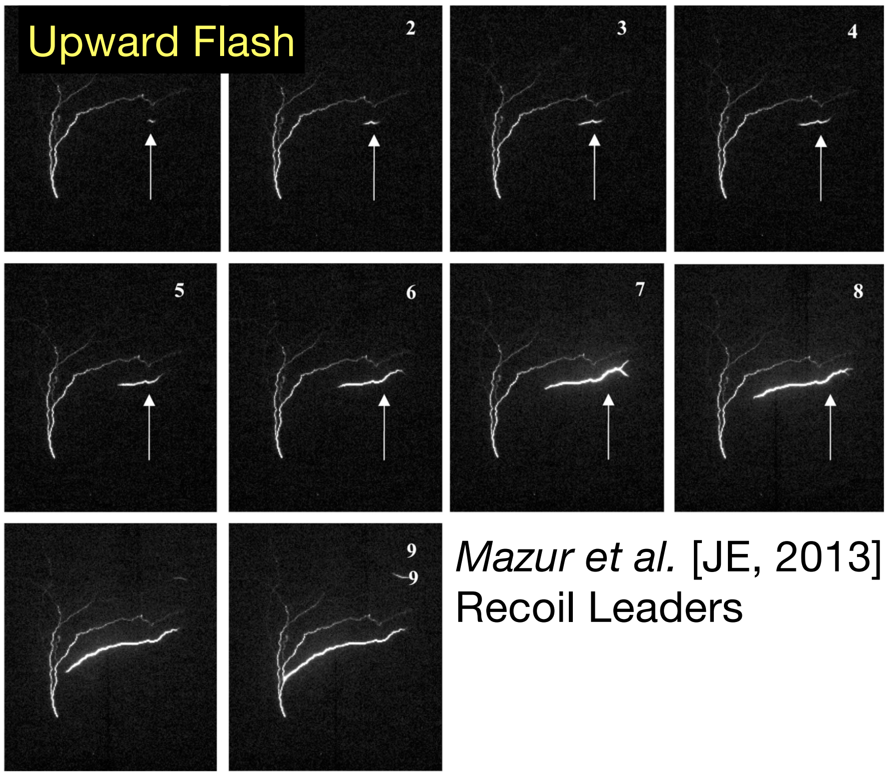
⭐ Work by NMT grad student John Pantuso



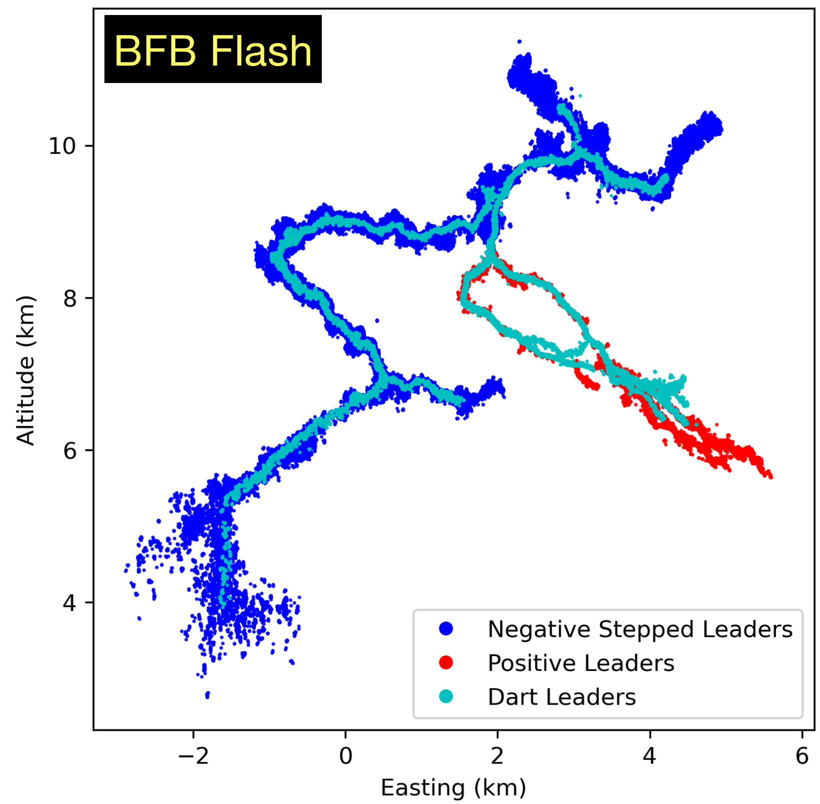
Dart Leaders

Dart Leaders (and their siblings)



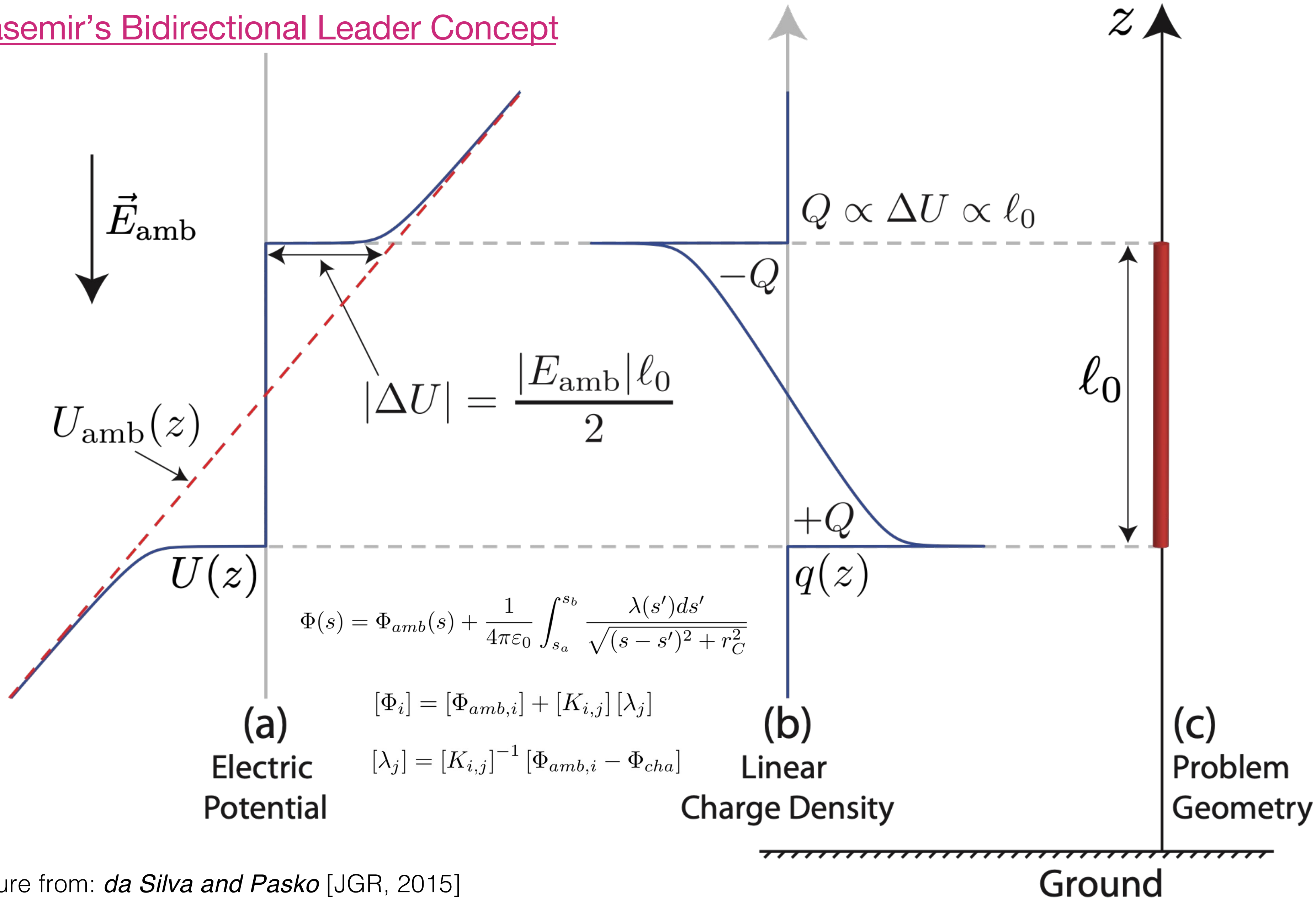


Mazur et al. [JE, 2013]
Recoil Leaders



Electrostatics of a Floating Leader

Kasemir's Bidirectional Leader Concept



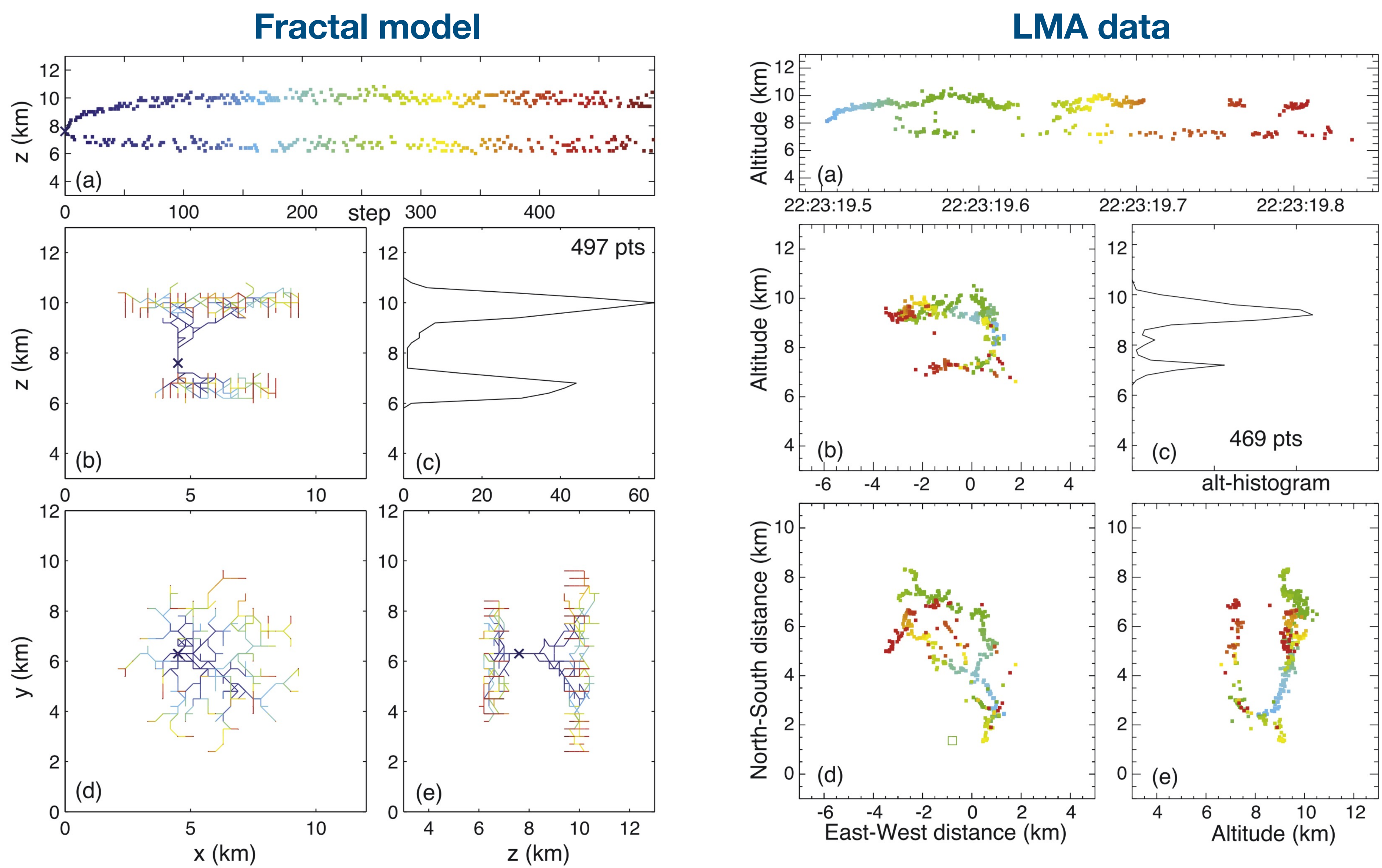
$$\Phi(s) = \Phi_{amb}(s) + \frac{1}{4\pi\varepsilon_0}\int_{s_a}^{s_b} \frac{\lambda(s')ds'}{\sqrt{(s-s')^2 + r_C^2}}$$

$$[\Phi_i] = [\Phi_{amb,i}] + [K_{i,j}]\,[\lambda_j]$$

$$[\lambda_j] = [K_{i,j}]^{-1}\,[\Phi_{amb,i} - \Phi_{cha}]$$

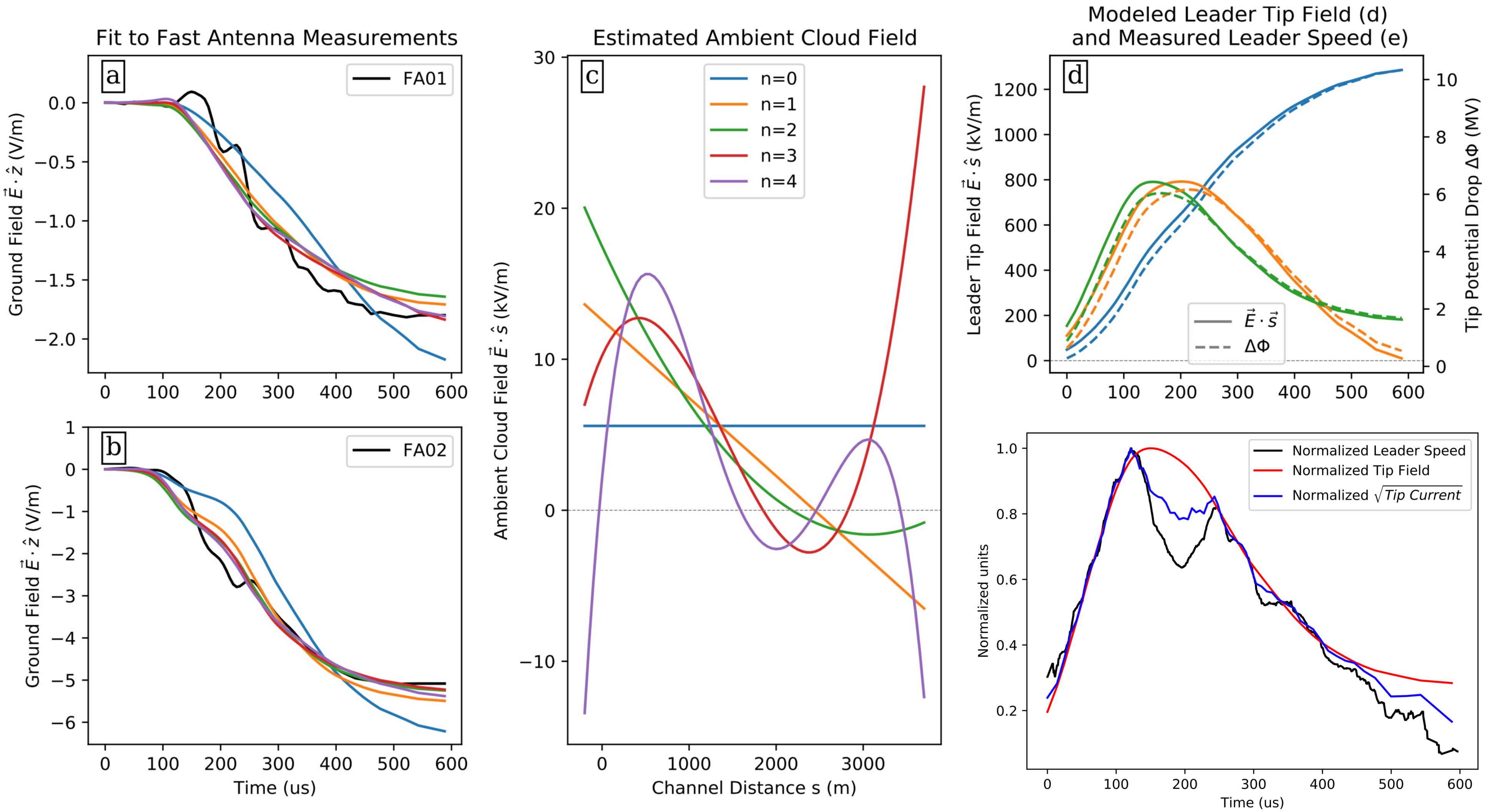
Picture from: *da Silva and Pasko* [JGR, 2015]

Interlude: 3D Leader Models are Electrostatic



Riousset et al. [JGR, 2007]

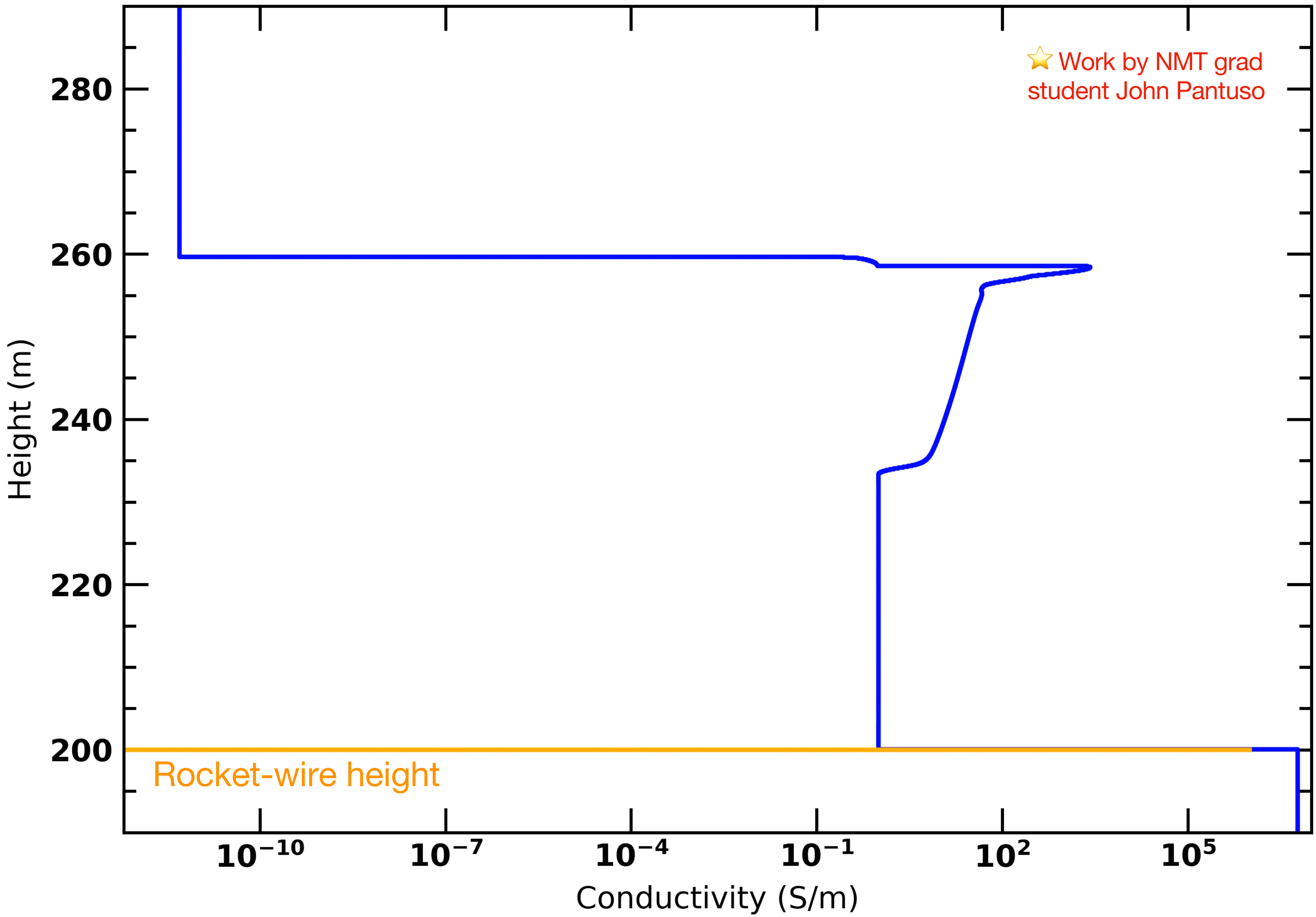
Probing Electrostatic Environment in Dart Leaders



⭐ Work by NMT grad student Daniel Jensen

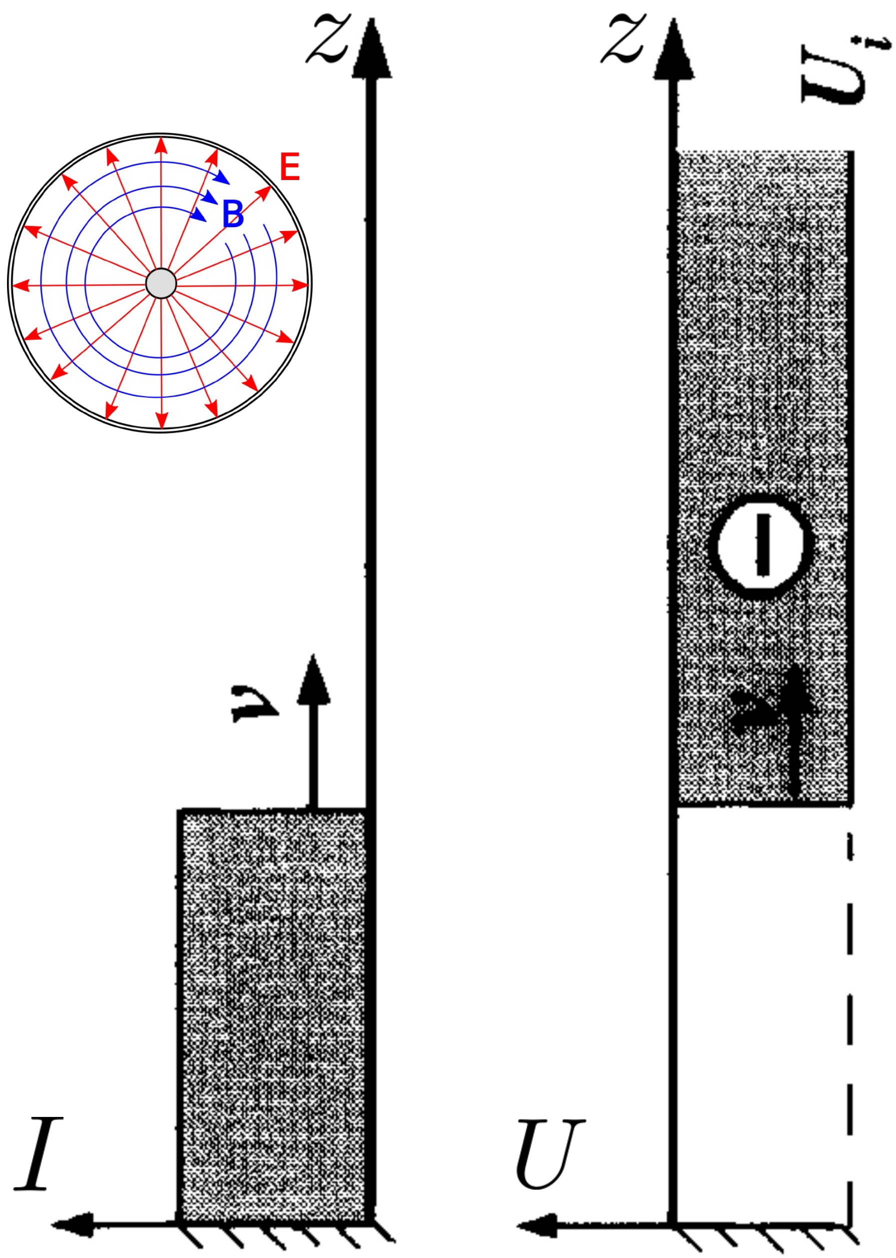
Jensen et al. [JGRA, 2024, Submitted for publication]

Spontaneous Emission of Dart Leaders



Return Stroke

Return Stroke is a Charge Neutralization Wave



Telegrapher's Equations:

$$L\frac{\partial I}{\partial t} + \frac{\partial U}{\partial z} + RI = 0$$

$$C\frac{\partial U}{\partial t} + \frac{\partial I}{\partial z} = 0$$

$$E_z = -\frac{\partial U}{\partial z} - \frac{\partial A}{\partial t} = RI$$

$$\frac{\partial \lambda}{\partial t} + \frac{\partial I}{\partial z} = 0$$

Bazelyan and Raizer [Lightning Physics and Protection]

Solution for a Perfectly-Conducting Channel

$$R = 0$$

$$L\frac{\partial I}{\partial t} + \frac{\partial U}{\partial z} = 0$$

$$C\frac{\partial U}{\partial t} + \frac{\partial I}{\partial z} = 0$$

$$\frac{\partial^2 I}{\partial t^2} + v^2\frac{\partial^2 I}{\partial z^2} = 0$$

General solution:

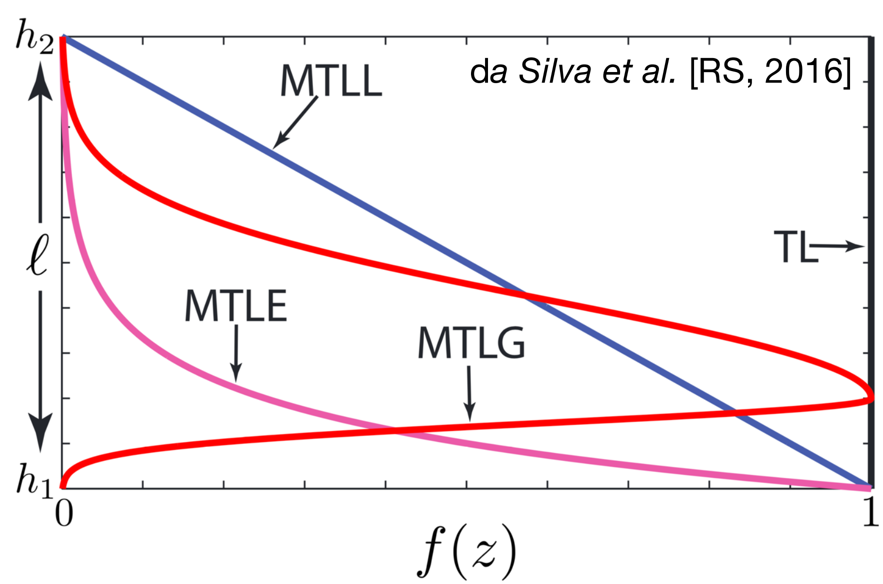
$$I(z,t) = i_1(z - vt) + i_2(z + vt)$$

$$U(z,t) = u_1(z - vt) + u_2(z + vt)$$

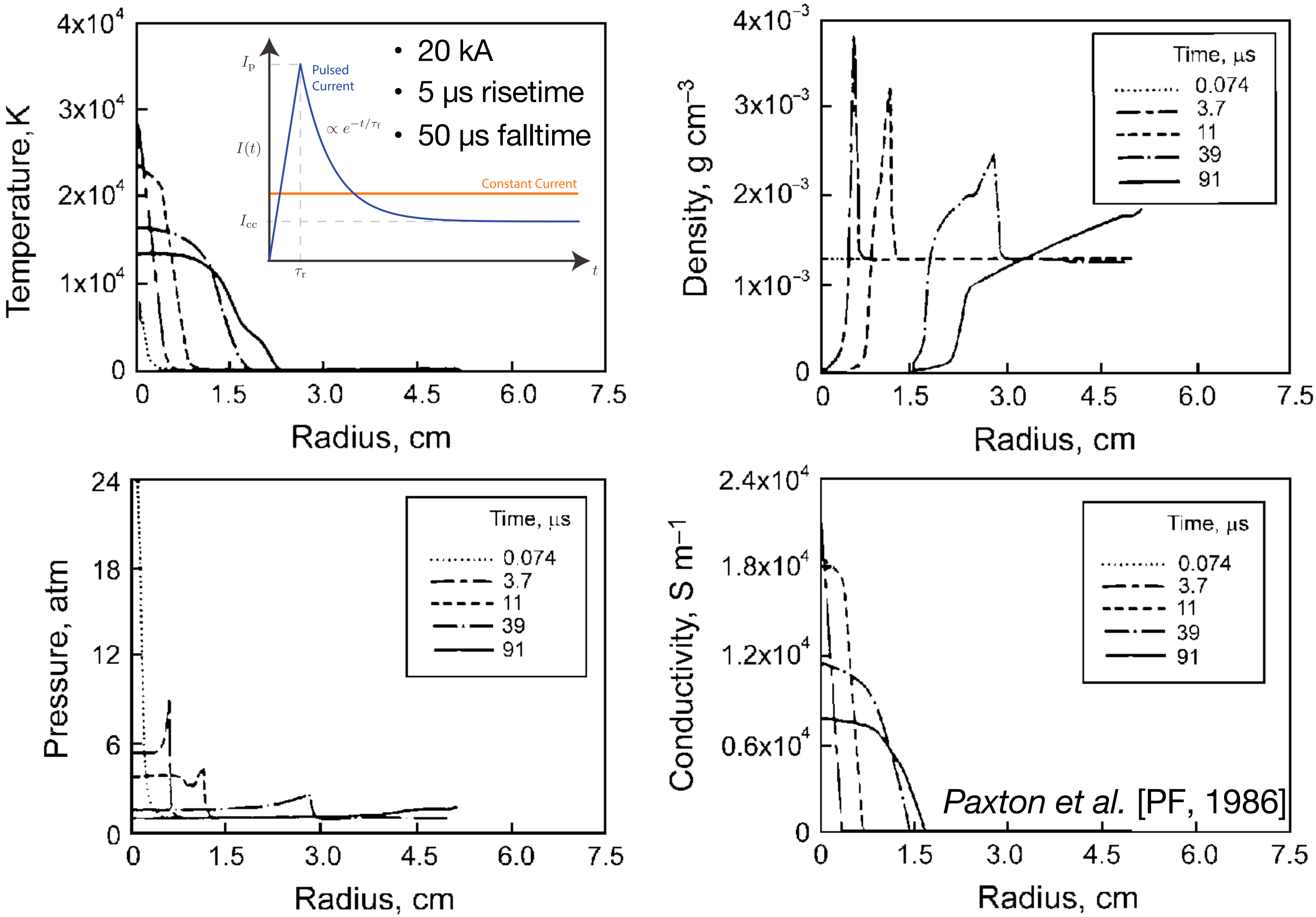
Rakov and Uman's "Engineering Models":

$$I(z,t) = f(z)I_{\mathrm{cb}}(t - z/v)$$

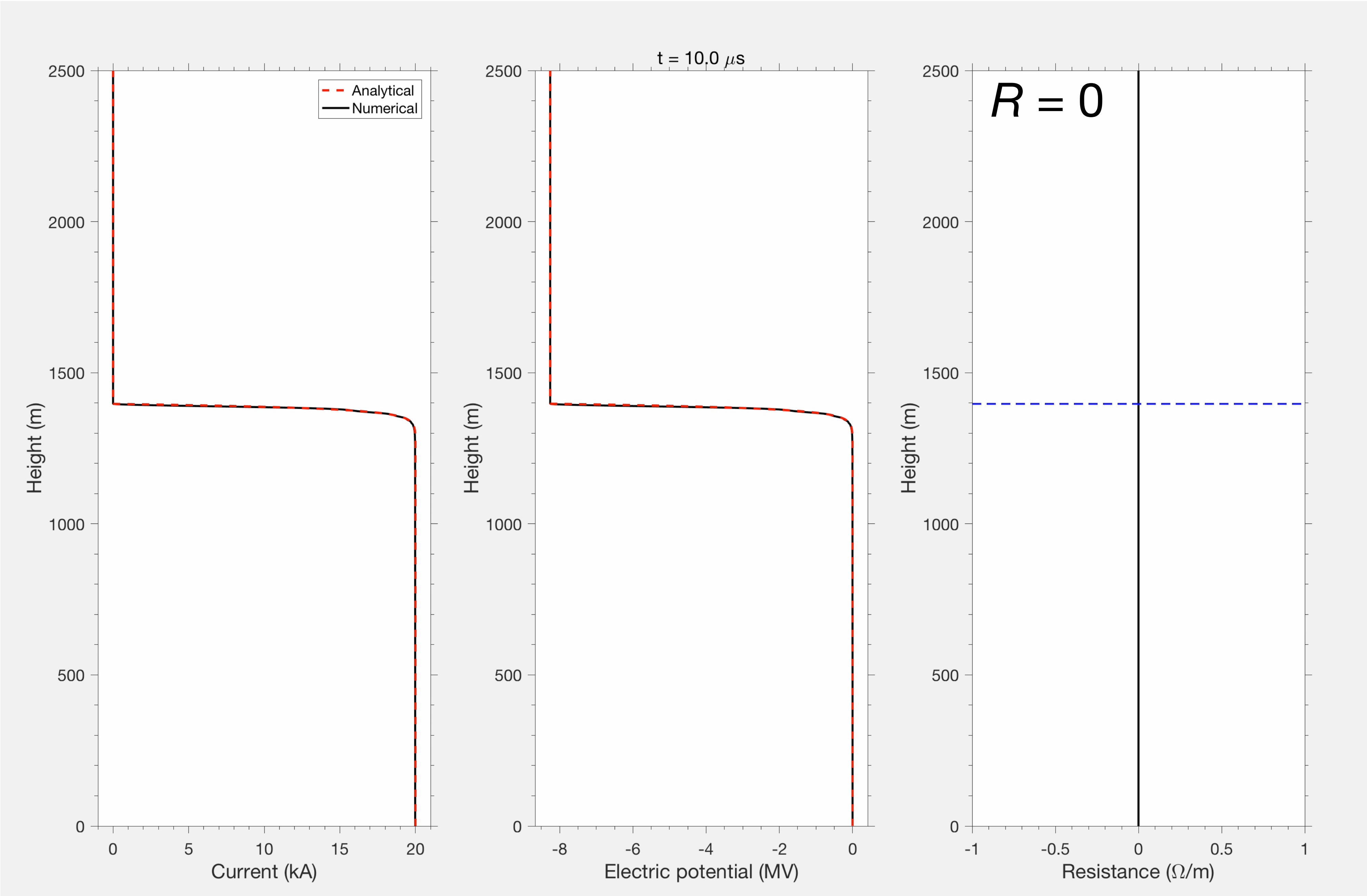
Rakov and Uman [Lightning: Physics and Effects]
Rakov and Uman [IEEE TEMC, 1998]



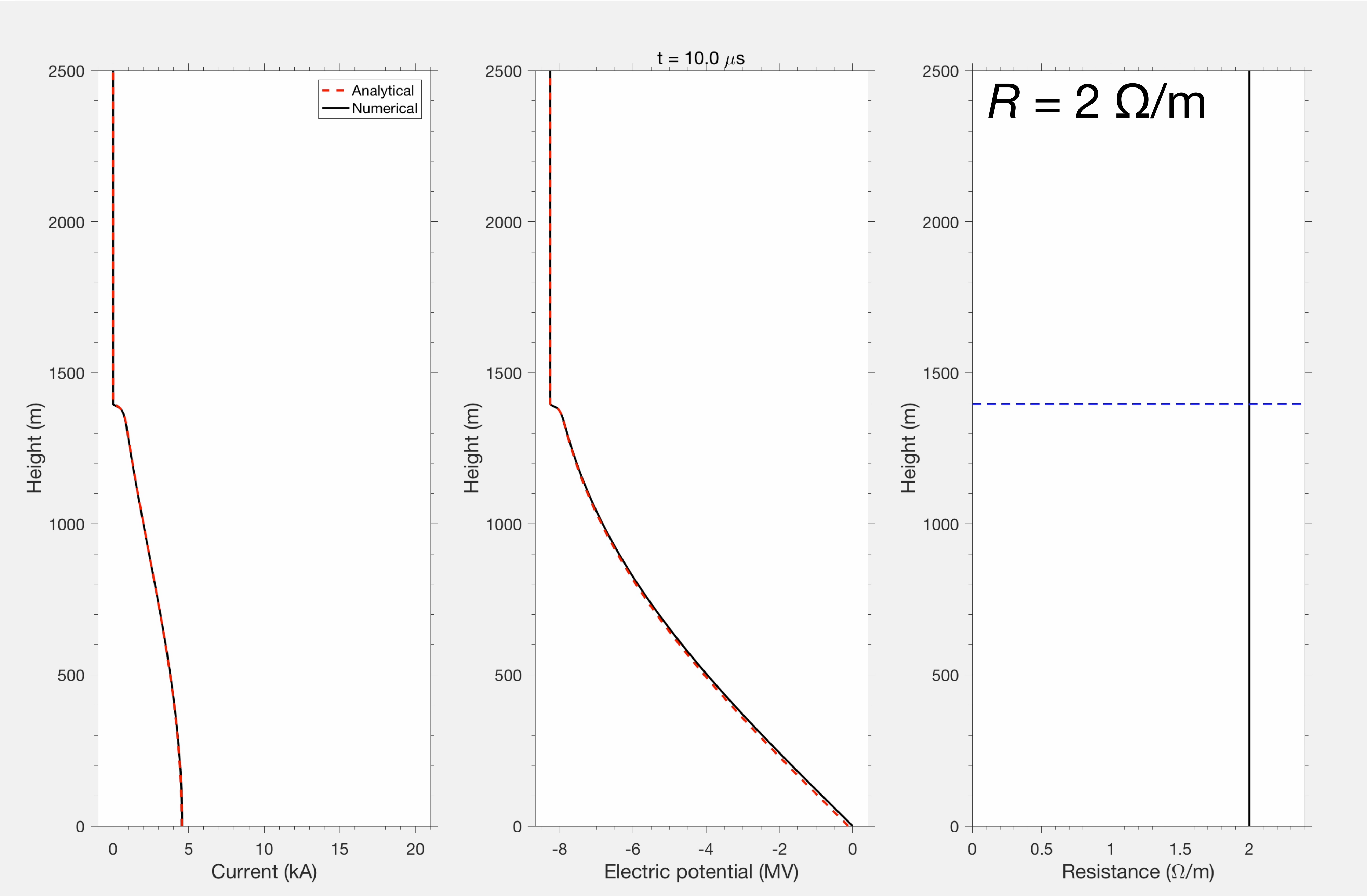
Channel is Transformed: Gas Dynamic Models



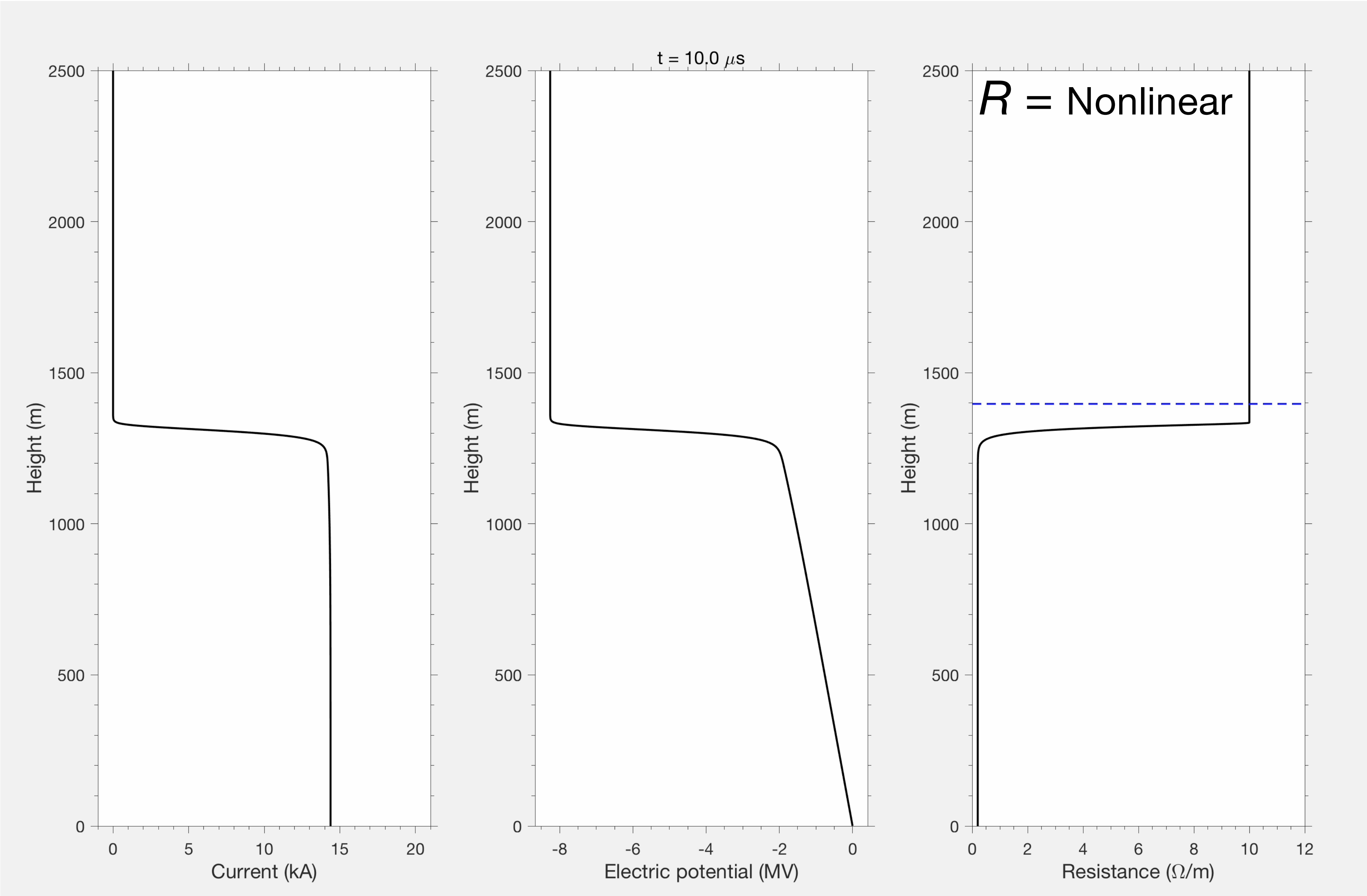
Return Stroke Current Wave Simulation



Return Stroke Current Wave Simulation



Return Stroke Current Wave Simulation



Plasma Negative Differential Resistance

Basic Nonlinear Resistance Model

Ohm's Law

$$E = RI = \frac{I}{\sigma \pi r_{\mathrm{c}}^2} = \frac{I}{e \mu_e n_e \pi r_{\mathrm{c}}^2}$$

Energy Balance

$$\rho_m c_p \frac{dT}{dt} = \eta_T \sigma E^2 - \frac{4\kappa_T}{r_{\mathrm{g}}^2}\left(T - T_{\mathrm{amb}}\right) - 4\pi\epsilon$$

Joule Heating; Heat Conduction; Radiation

Electrons

$$\frac{dn_e}{dt} = \left(\nu_{\mathrm{i}} - \nu_{\mathrm{a2}} - \nu_{\mathrm{a3}}\right) n_e + \nu_{\mathrm{d}} n_n + k_{ep} n_{\mathrm{LTE}}^2 - k_{ep} n_e \left(n_e + n_n\right)$$

Ionization; 2-Body Attachment; 3-Body Attachment; Detachment; Thermal Ionization; Electron-Ion Recombination

Negative Ions

$$\frac{dn_n}{dt} = \left(\nu_{\mathrm{a2}} + \nu_{\mathrm{a3}}\right) n_e - \nu_{\mathrm{d}} n_n - k_{np} n_n \left(n_e + n_n\right)$$

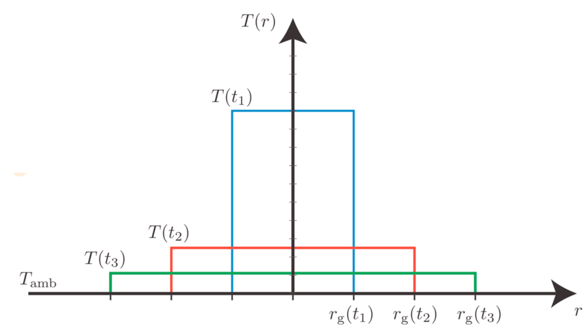
Positive-Negative Ion Recombination

$$\frac{dr_{\mathrm{c}}^2}{dt} = 4D_{\mathrm{a}}$$

Expansion of electrodynamic radius

$$\frac{dr_{\mathrm{g}}^2}{dt} = \frac{4\kappa_T}{\rho_m c_p}$$

Expansion of thermal radius



da Silva et al. [JGRA, 2019]

Basic Nonlinear Resistance Model

Ohm's Law $E = RI = \frac{I}{\sigma \pi r_\mathrm{c}^2} = \frac{I}{e \mu_e n_e \pi r_\mathrm{c}^2}$

Energy Balance $\rho_m c_p \frac{dT}{dt} = \eta_T \sigma E^2 - \frac{4\kappa_T}{r_\mathrm{g}^2}(T - T_\mathrm{amb}) - 4\pi\epsilon$

Joule Heating — Heat Conduction — Radiation

Electrons $\frac{dn_e}{dt} = (\nu_\mathrm{i} - \nu_\mathrm{a2} - \nu_\mathrm{a3}) n_e + \nu_d n_n + k_{ep} n_\mathrm{LTE}^2 - k_{ep} n_e (n_e + n_n)$

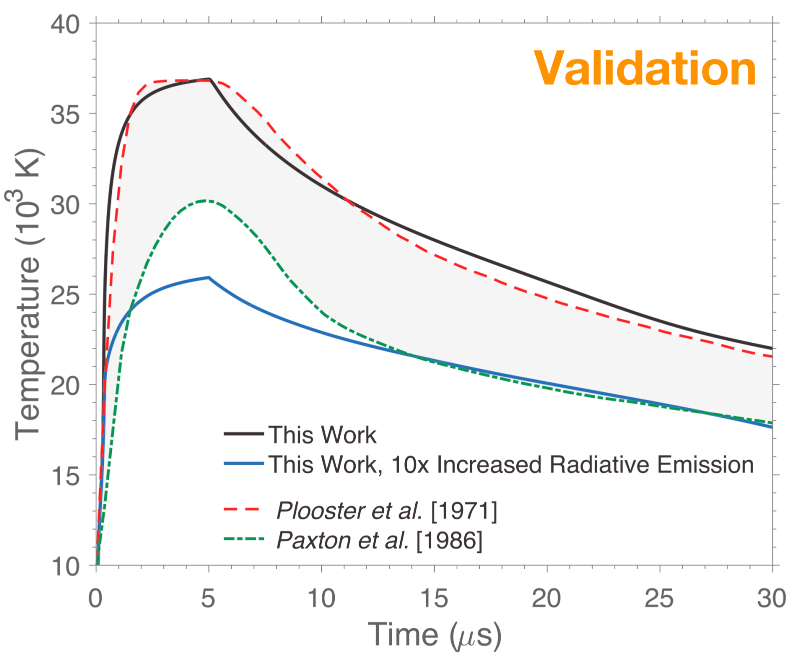
Ionization — 3-Body Attachment — Detachment — Thermal Ionization — Electron-Ion Recombination

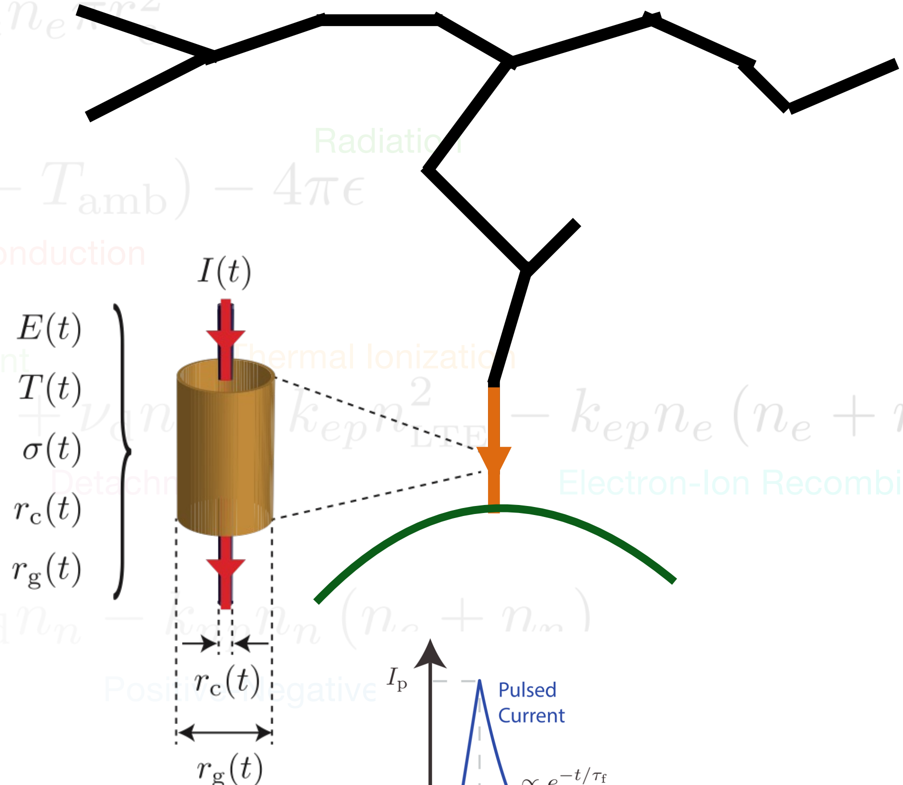
Negative Ion $\ldots - \nu_d n_n - k_{ep} n_n (n_e + n_n)$

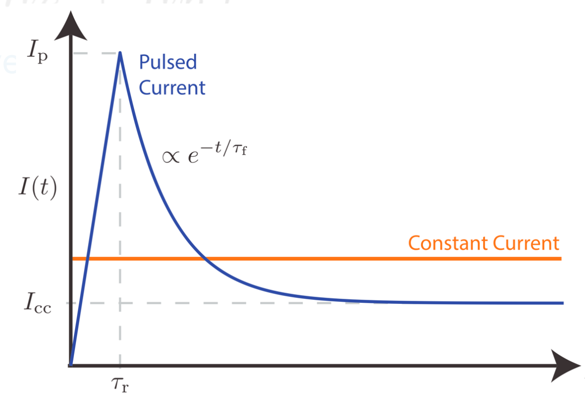
Positive-negative

hydrodynamic radius

thermal radius



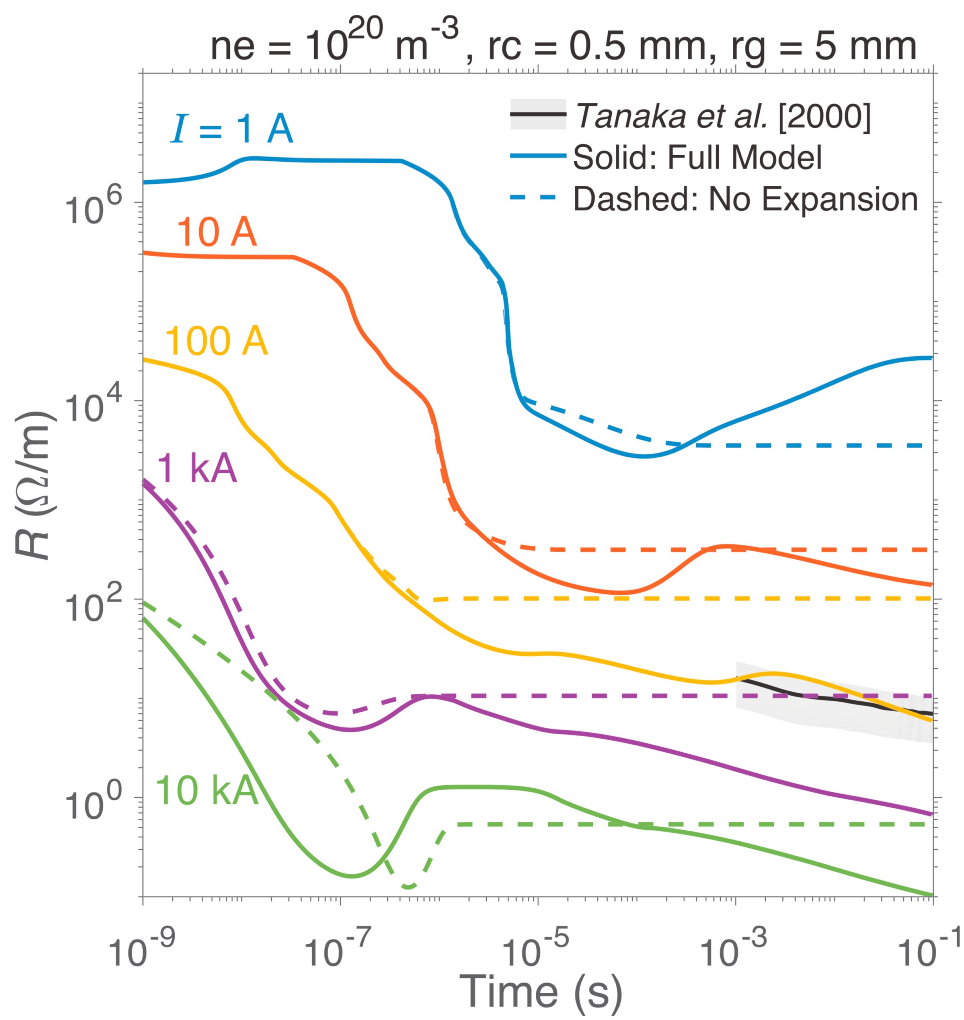




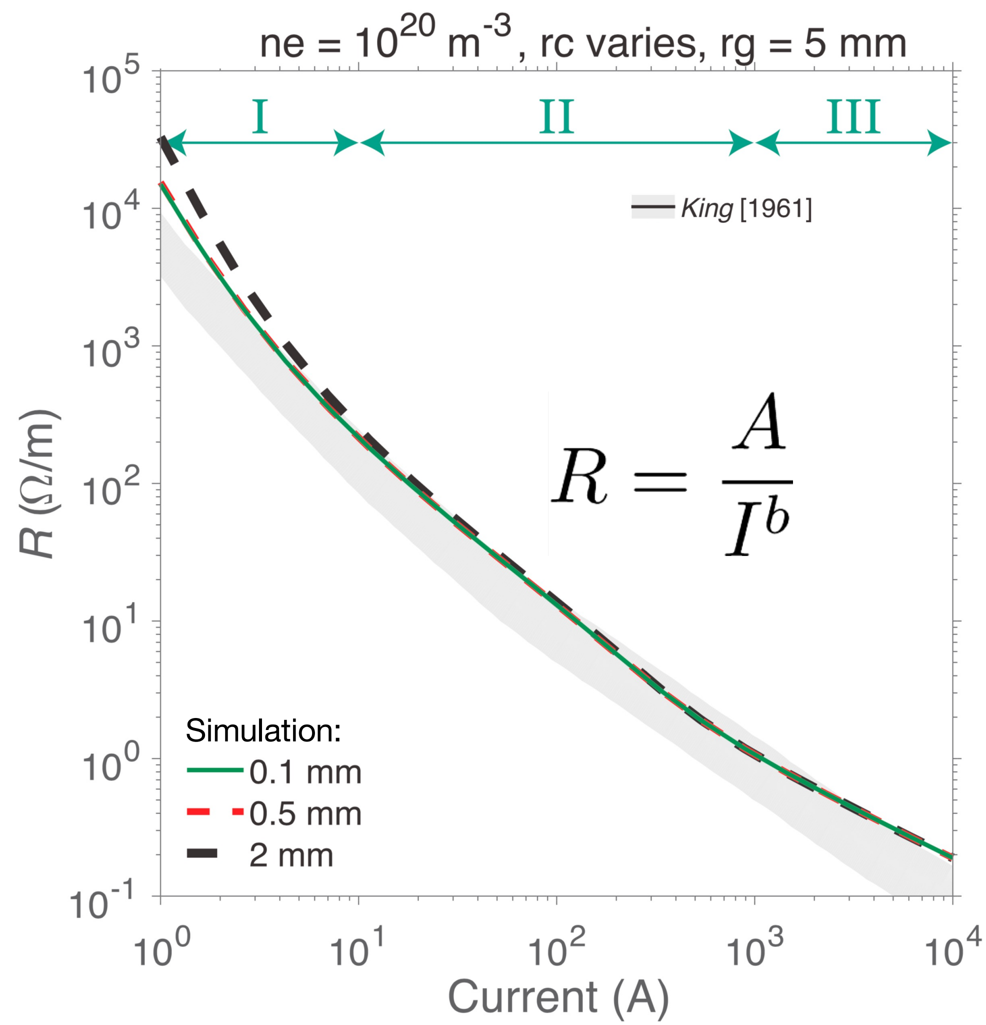
da Silva et al. [JGR, 2019]

Negative Differential Resistance

Resistance change as a function of time



Steady-state resistance at 10 ms



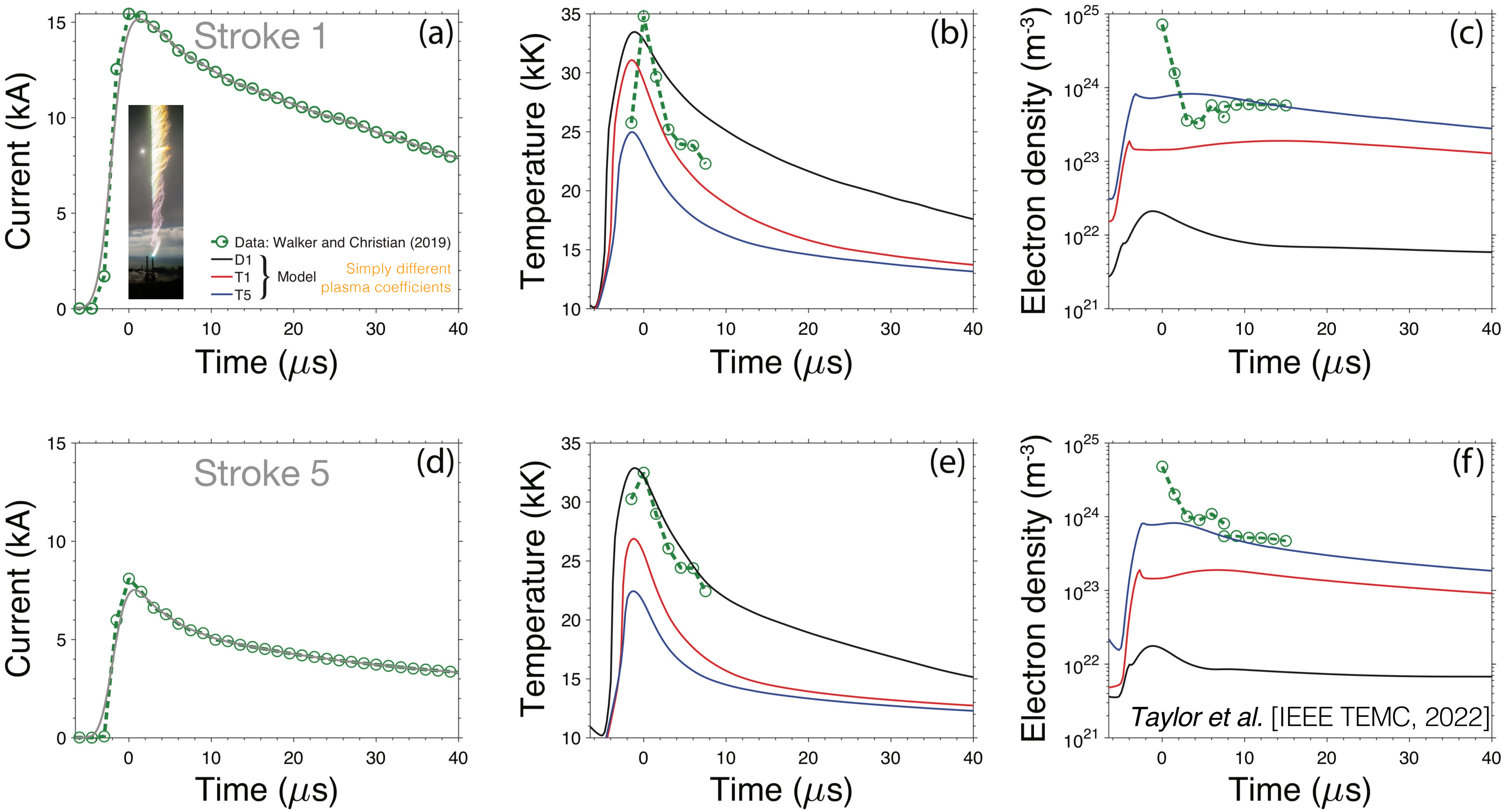
$$R = \frac{A}{I^b}$$

In the 3 different regions, the steady state is given by a balance between Joule heating and:

I. Thermal Conduction
II. Both I and III
III. Radiative Emission

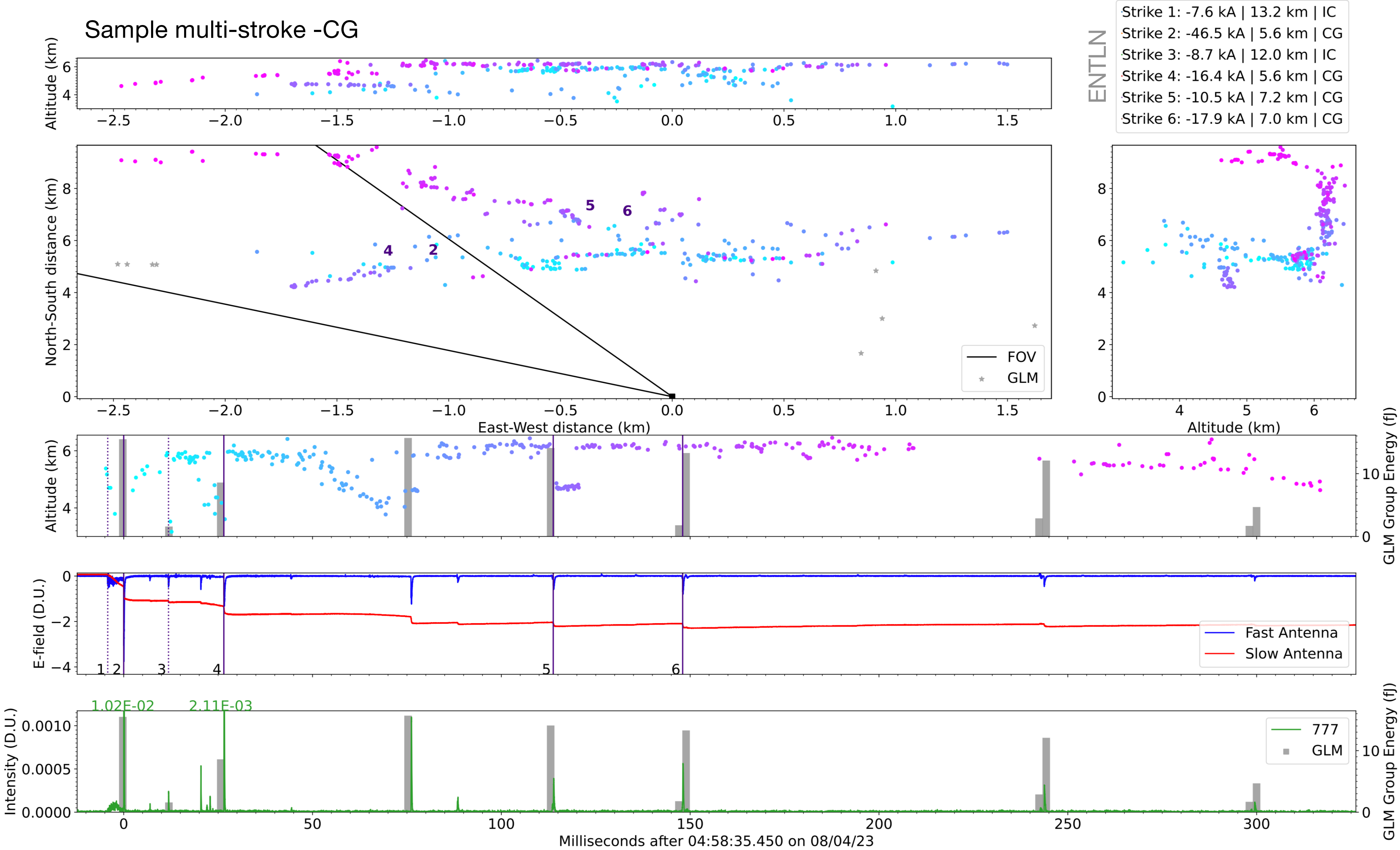
da Silva et al. [JGRA, 2019]

Rocket-Triggered Lightning: Great for Model Validation



- Model is driven by measured channel-base current and calculated temperature and electron density are compared to spectroscopic measurements from *Walker and Christian* [JGR, 2017; 2019].

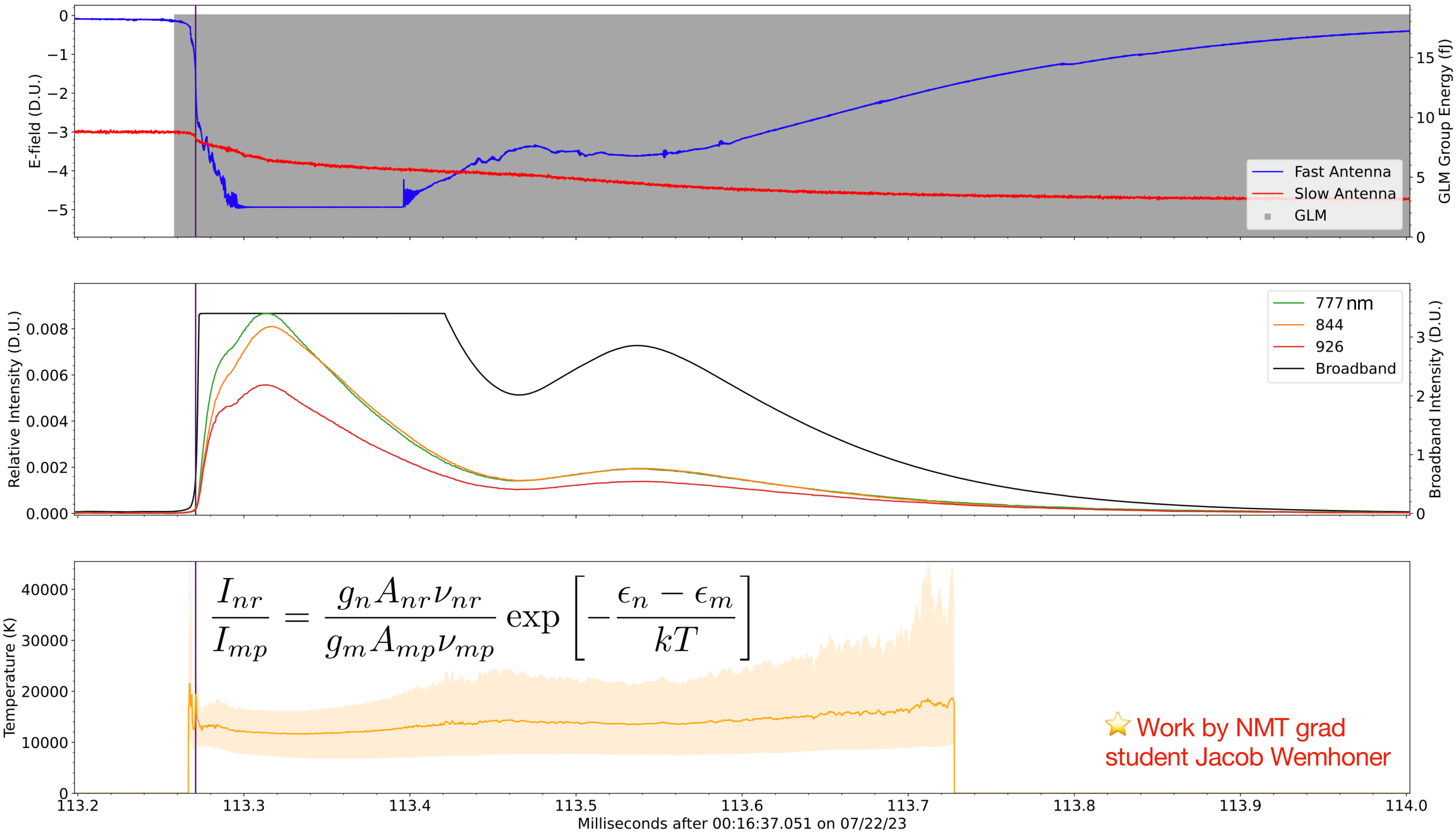
Multi-band, Multi-platform Observations



Question tip: ask me later about a +CG example!

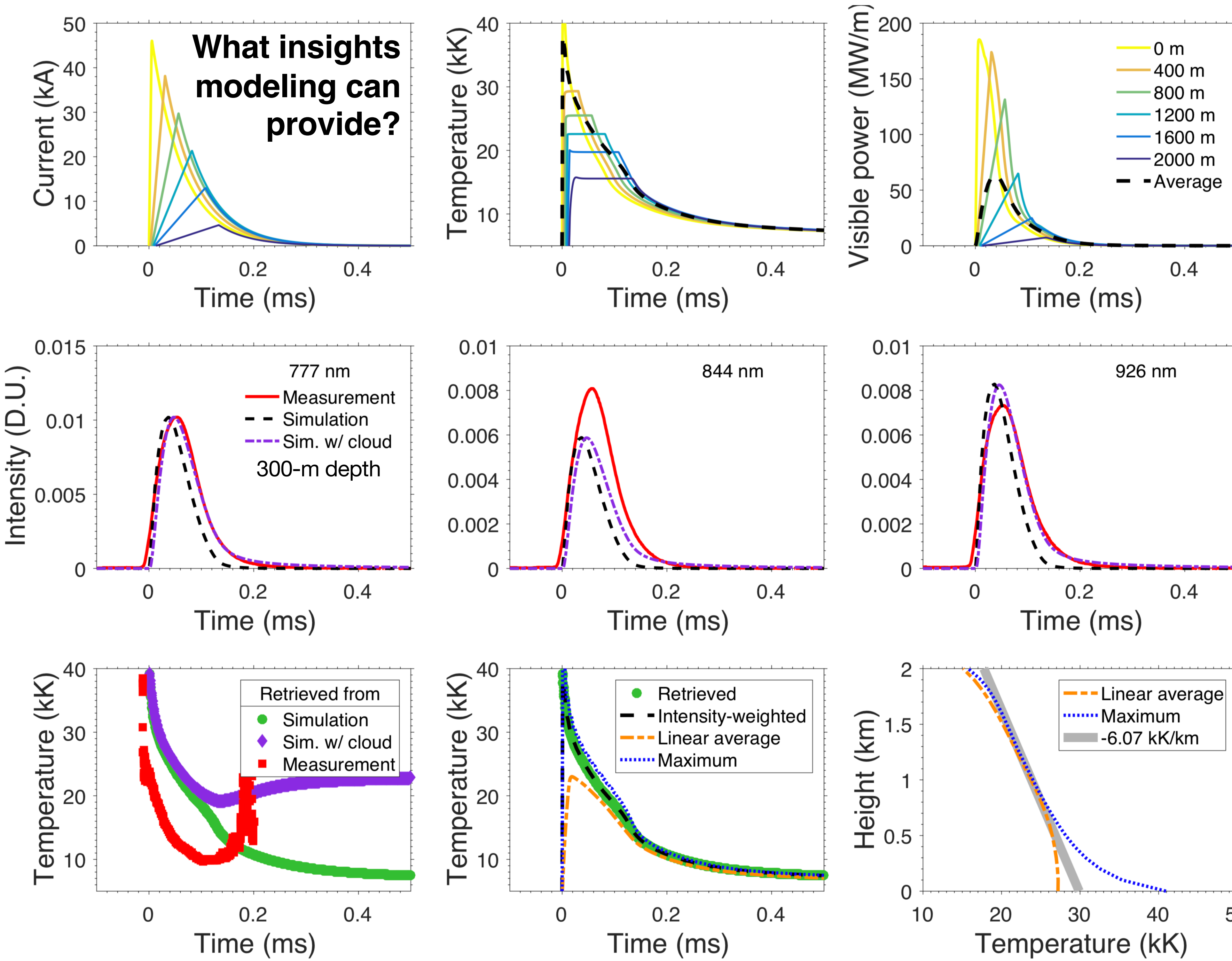
Atomic Oxygen Photometric Temperature

Sample -CG return stroke



$$\frac{I_{nr}}{I_{mp}} = \frac{g_n A_{nr} \nu_{nr}}{g_m A_{mp} \nu_{mp}} \exp\left[-\frac{\epsilon_n - \epsilon_m}{kT}\right]$$

⭐ Work by NMT grad student Jacob Wemhoner


What insights modeling can provide?
Current (kA)
Time (ms)
Temperature (kK)
Visible power (MW/m)
0 m
400 m
800 m
1200 m
1600 m
2000 m
Average
Intensity (D.U.)
777 nm
Measurement
Simulation
Sim. w/ cloud
300-m depth
844 nm
926 nm
Retrieved from
Simulation
Sim. w/ cloud
Measurement
Retrieved
Intensity-weighted
Linear average
Maximum
Height (km)
Linear average
Maximum
-6.07 kK/km

Summary

Summary

1. Models for leaders and return stroke exist, but they are often focused on particular aspects of the problem.

2. Need continued development, integration, & validation.

3. NMT effort is highly student driven. There are students in training now that can join the workforce later.

4. Model validation studies need to be intentionally designed. Langmuir Lab is an ideal place for such.

5. There is additional work by international colleagues that can be leveraged (e.g., in Russia, Europe).

Lightning Physics Session at AGU24

Physics of Streamers, Leaders, and the Lightning Discharge

- Session has been proposed and we are waiting on AGU evaluation.
- Great avenue to advertise this workshop's efforts (we could dedicate an invited talk).
- 9-13 December, 2024, Washington, DC.

AGU FALL MEETING

Questions?

Acknowledgements:

This work has been supported by NSF CAREER award AGS-2046043 to New Mexico Tech.

Caitano L. da Silva, J. G. Pantuso, J. Wemhoner, D. Jensen, A. F. R. Leal, R. G. Sonnenfeld

LANGMUIR LABORATORY FOR ATMOSPHERIC RESEARCH

Department of Physics & Langmuir Lab
New Mexico Tech, Socorro, NM 87801
Email contact: caitano.dasilva@nmt.edu

NEW MEXICO TECH
SCIENCE · ENGINEERING · RESEARCH · UNIVERSITY

Extra Slides: Misc.

More Multi-platform Observations

BIMAP-3D Lightning Interferometer and GLM

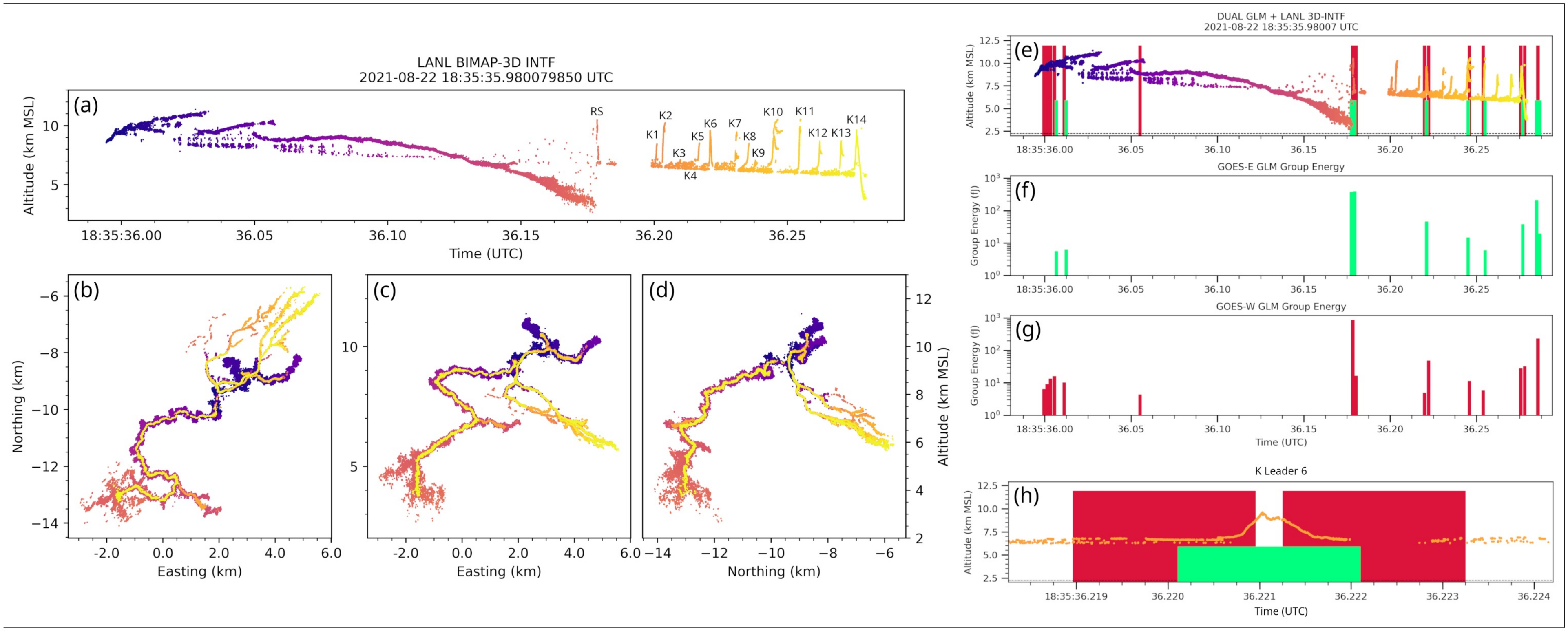
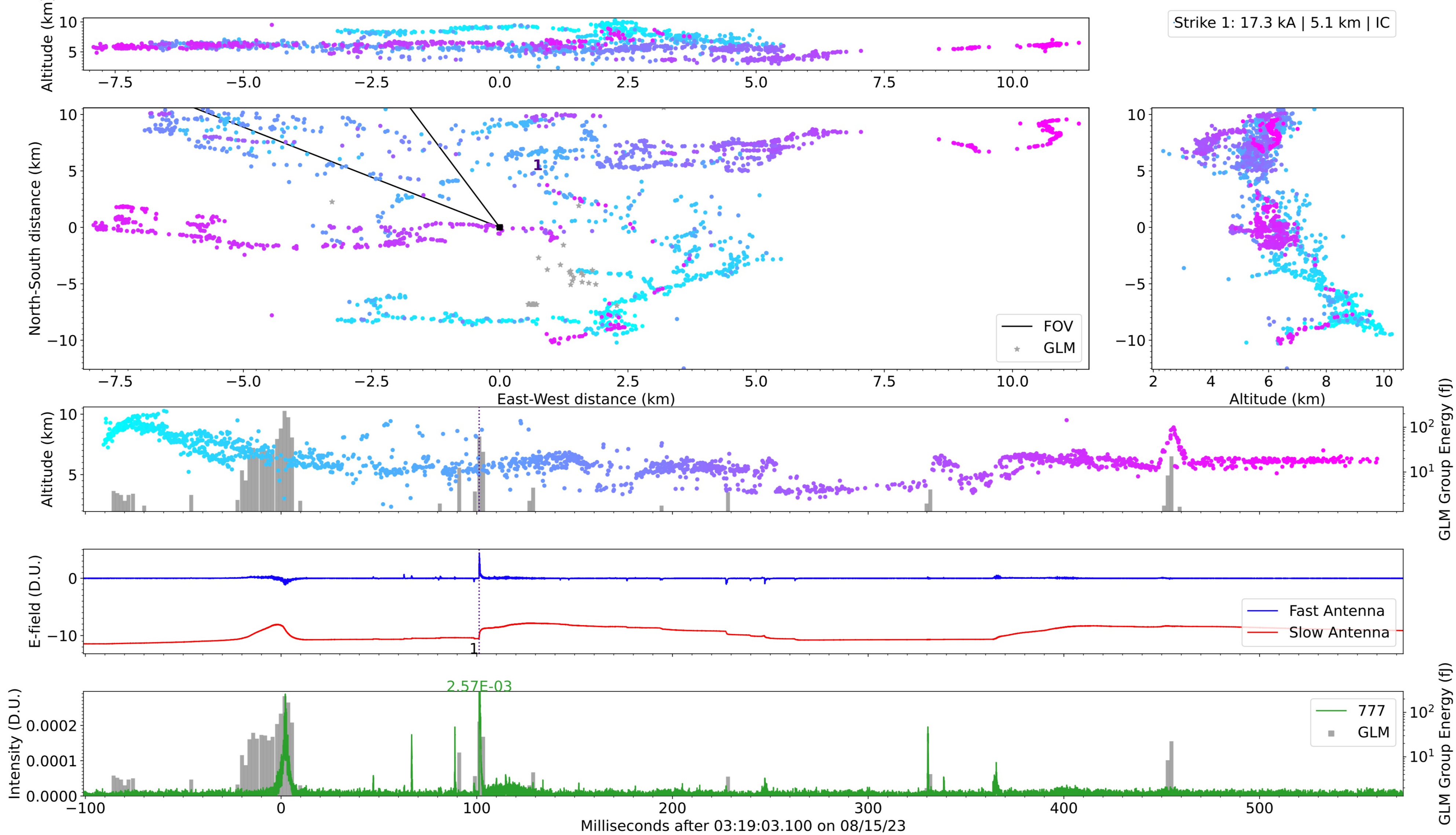


Figure 5: A flash captured by the Los Alamos BIMAP-3D lightning interferometer[3], GLM, and Earth Networks Total Lightning Network (ENTLN). GLM detects some portions of stepped leaders, the return stroke (RS) K leaders (K1-K14) and a dart leader (K14) The detection times of GLM East and GLM West groups (vertical bars in panels "e" & "h") are offset.

⭐ Work by NMT grad student R. Stetson Reger

Multi-band, Multi-platform Observations

Sample **+CG**



Streamer-to-Leader Transition Model

[*da Silva and Pasko*, JGR, 118, 13561, 2013]

- Dynamics of neutral gas:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{\mathrm{v}}) = 0$$ Conservation of Mass

$$\frac{\partial \rho\vec{\mathrm{v}}}{\partial t} + \vec{\nabla}\cdot(\rho\vec{\mathrm{v}}\vec{\mathrm{v}}) = -\vec{\nabla}p + \frac{1}{3}\mu\vec{\nabla}(\vec{\nabla}\cdot\vec{\mathrm{v}}) + \mu\nabla^2\vec{\mathrm{v}}$$ Momentum

$$\frac{\partial \varepsilon}{\partial t} + \vec{\nabla}\cdot[(\varepsilon + p)\vec{\mathrm{v}}] = Q_{\mathrm{T}}^{\mathrm{eff}} + \vec{\nabla}\cdot(\kappa_{\mathrm{T}}^{*}\vec{\nabla}T)$$ Translational Energy

$$\frac{\partial \varepsilon_{\mathrm{V}}}{\partial t} + \vec{\nabla}\cdot(\varepsilon_{\mathrm{V}}\vec{\mathrm{v}}) = Q_{\mathrm{V}}^{\mathrm{eff}} + \vec{\nabla}\cdot(D_{\mathrm{V}}\vec{\nabla}\varepsilon_{\mathrm{V}})$$ Vibrational Energy

- Comprehensive plasma chemistry:

$$\frac{\partial n_j}{\partial t} + \vec{\nabla}\cdot(n_j\vec{\mathrm{v}}) = S_j + \vec{\nabla}\cdot(D_j\vec{\nabla}n_j)$$

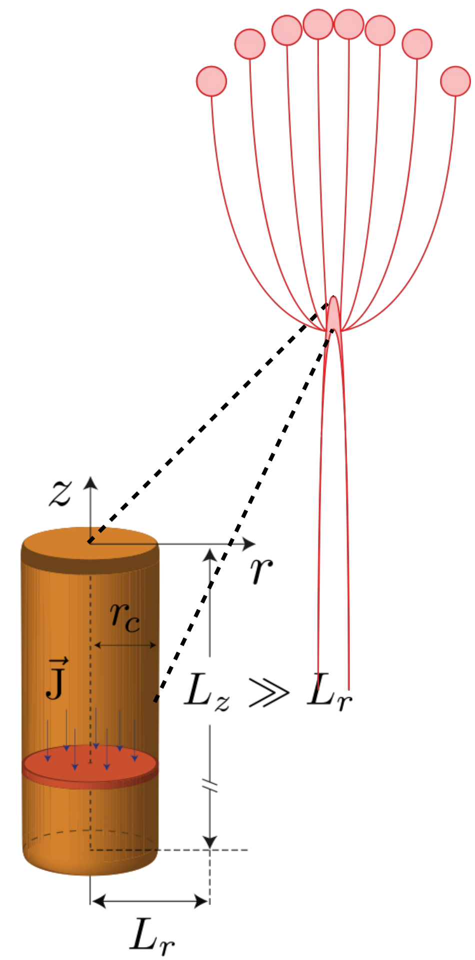
21 Species
106 Reactions

- Energy exchange between charged and neutral particles:

$$\frac{\partial \varepsilon}{\partial t} \propto Q_{\mathrm{T}}^{\mathrm{eff}} = Q_{\mathrm{T}} + Q_{\mathrm{L}} + Q_i + Q_{\mathrm{VT}} + Q_{\mathrm{VV}} + Q_{\mathrm{D}}$$

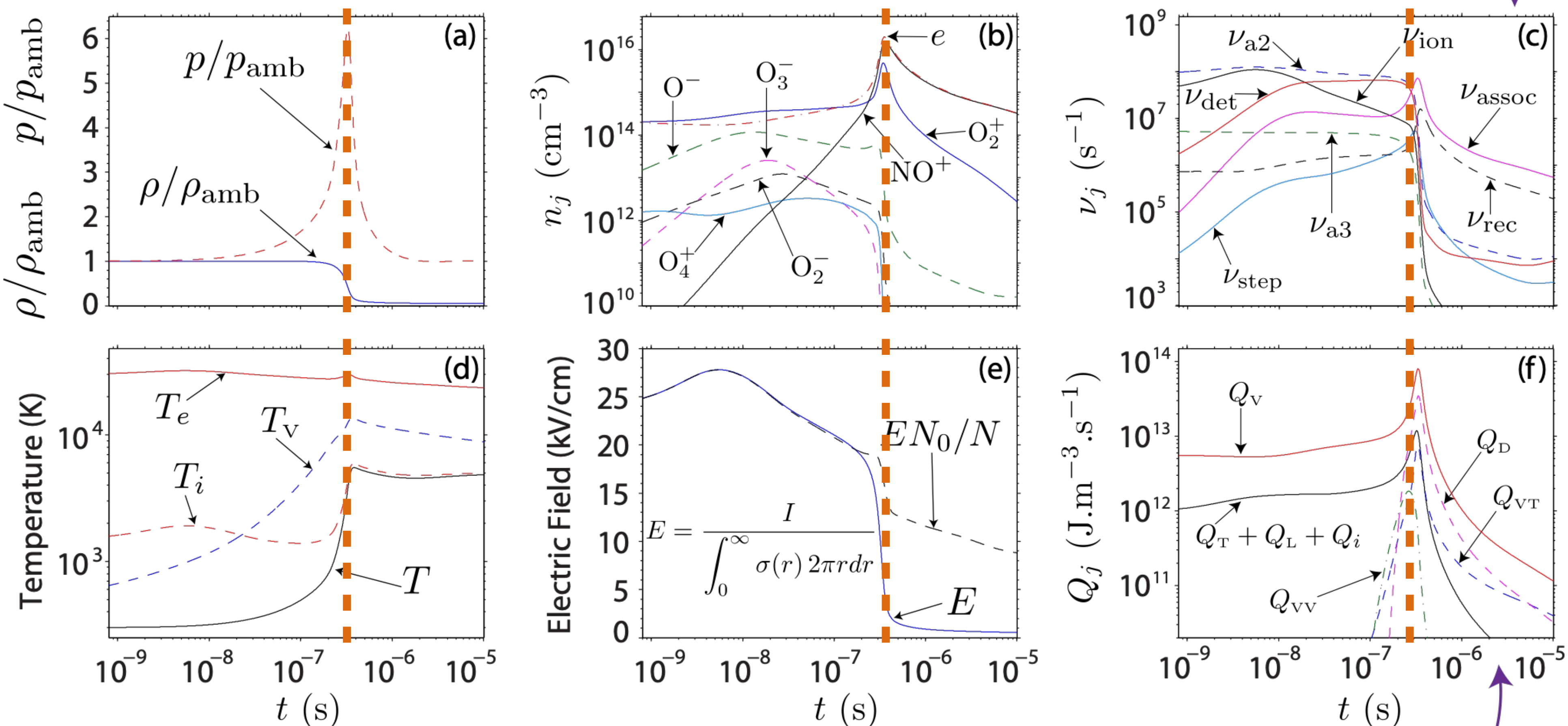
$$\frac{\partial \varepsilon_{\mathrm{V}}}{\partial t} \propto Q_{\mathrm{V}}^{\mathrm{eff}} = Q_{\mathrm{V}} - Q_{\mathrm{VT}} - Q_{\mathrm{VV}} - 2Q_{\mathrm{D}}$$

Fast air heating vs.
Delayed vibrational relaxation



Streamer-to-Leader Transition: Temporal Dynamics

$$\frac{\partial n_e}{\partial t} \propto S_e = (\nu_{\rm ion} + \nu_{\rm step} + \nu_{\rm assoc} + \nu_{\rm det} - \nu_{\rm a2} - \nu_{\rm a3} - \nu_{\rm rec})\, n_e$$

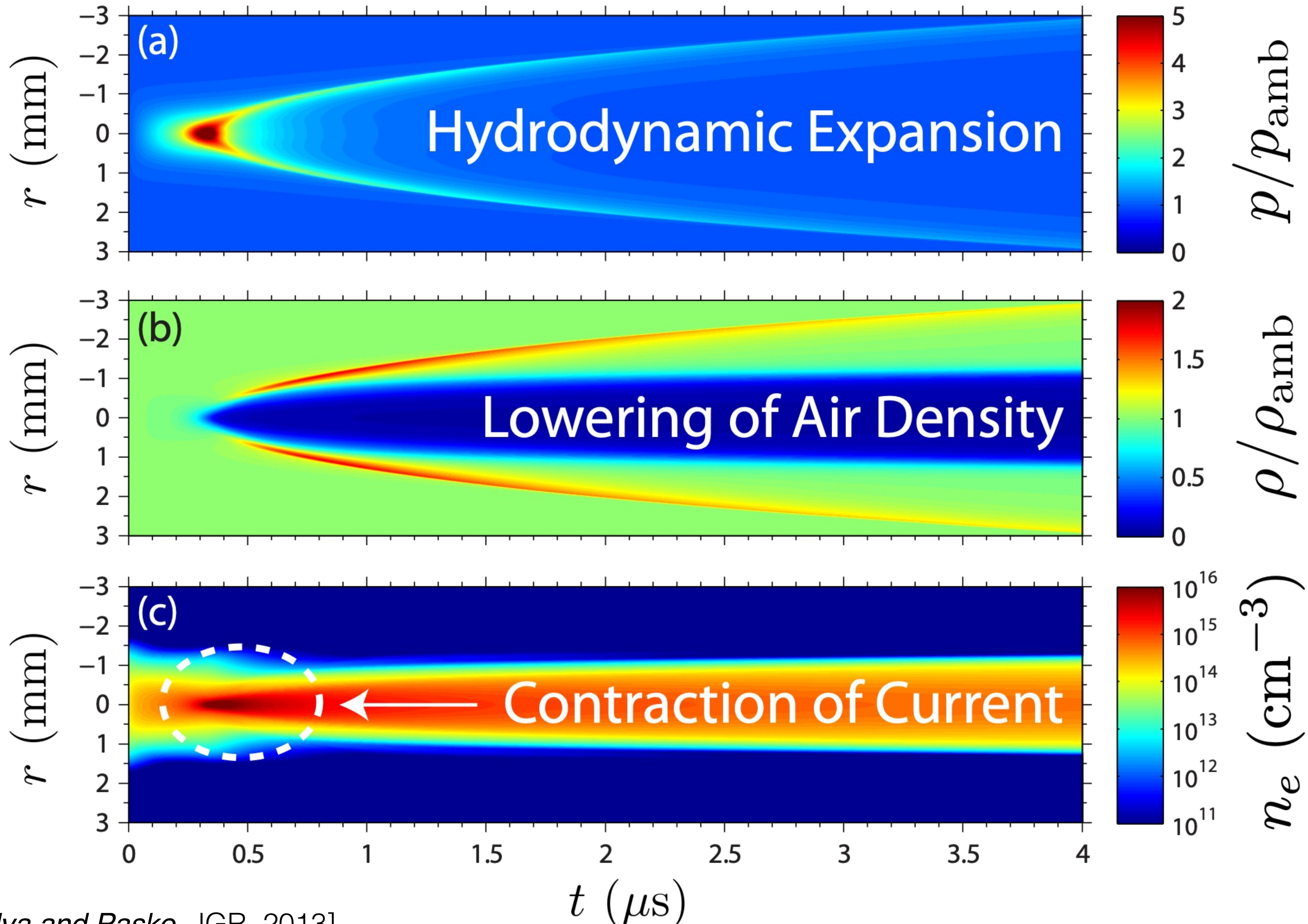


$$\frac{\partial T}{\partial t} \propto Q_{\rm T}^{\rm eff} = Q_{\rm T} + Q_{\rm L} + Q_i + Q_{\rm VT} + Q_{\rm VV} + Q_{\rm D}$$

Current = const. = 1 A; Pressure = 1 atm

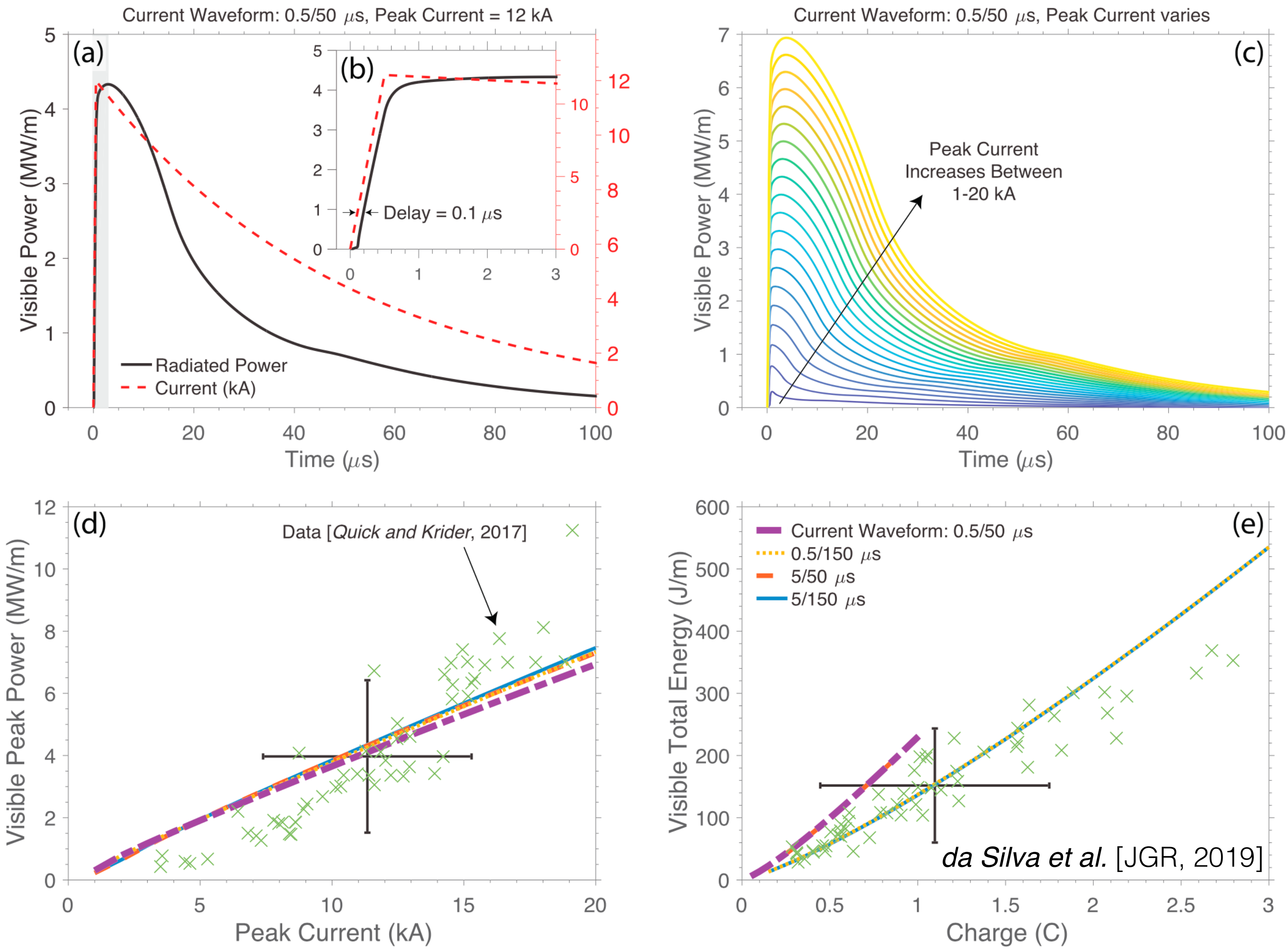
[*da Silva and Pasko*, JGR, 2013]

Streamer-to-Leader Transition: Radial Dynamics



[*da Silva and Pasko*, JGR, 2013]

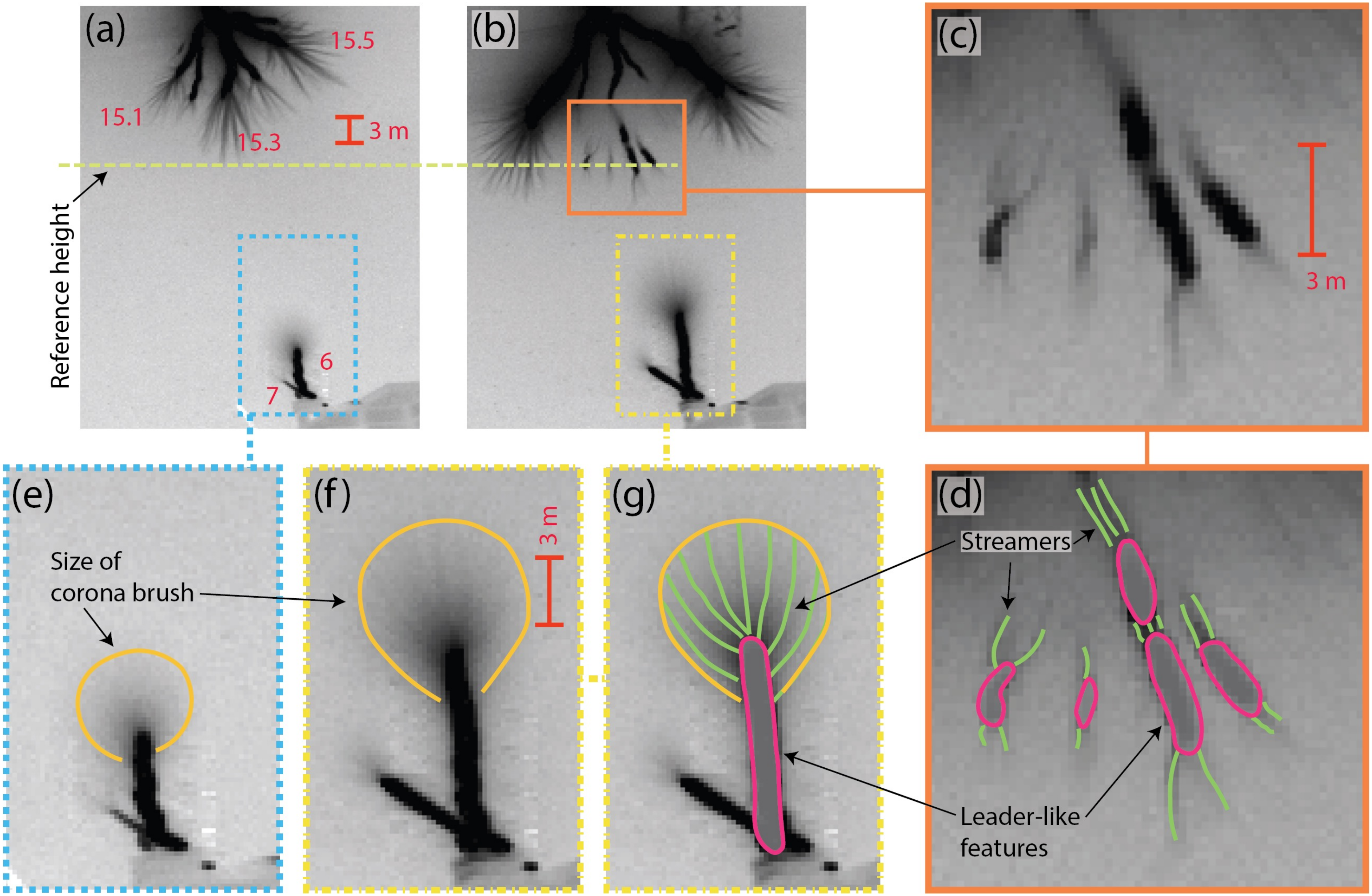
Optical Power Radiated by Lightning



Lightning Attachment to Residential Buildings

Saba et al. [GRL, 2022], 40 kfps video, ~200 m distance

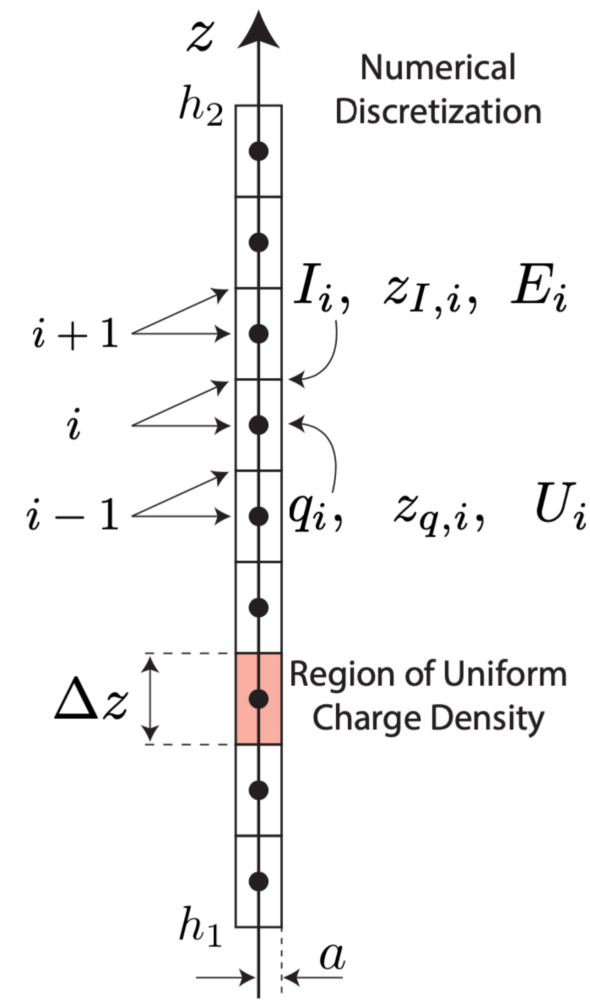
Streamer Zones



Saba et al. [GRL, 2022]

Electrodynamic Model of Leaders

- $$\begin{cases} U(z,t) = U_{\text{amb}}(z) + \dfrac{1}{4\pi\varepsilon}\displaystyle\int_{h_1}^{h_2} \dfrac{q(z',t')}{R(z,z')}\mathrm{d}z' \\ A(z,t) = \dfrac{\mu}{4\pi}\displaystyle\int_{h_1}^{h_2} \dfrac{I(z',t')}{R(z,z')}\mathrm{d}z' \\ \dfrac{\partial A}{\partial t} + \dfrac{\partial U}{\partial z} + \dfrac{I}{G} = 0 \\ \dfrac{\partial q}{\partial t} + \dfrac{\partial I}{\partial z} = 0 \end{cases}$$
- $R(z,z') = \sqrt{(z-z')^2 + a^2}$ and $t' = t - R(z,z')/c$.
- $\varepsilon = 5.3\varepsilon_0$ and $\mu = \mu_0$ [*Moini et al.*, JGR, 105, D24, 2000].
- Equations are solved with method of moments applied to time-domain antenna theory [e.g., *Miller et al.*, JCP, 12(1), 24, 1973; *Carlson et al.*, JGR, 115, A10324, 2010].



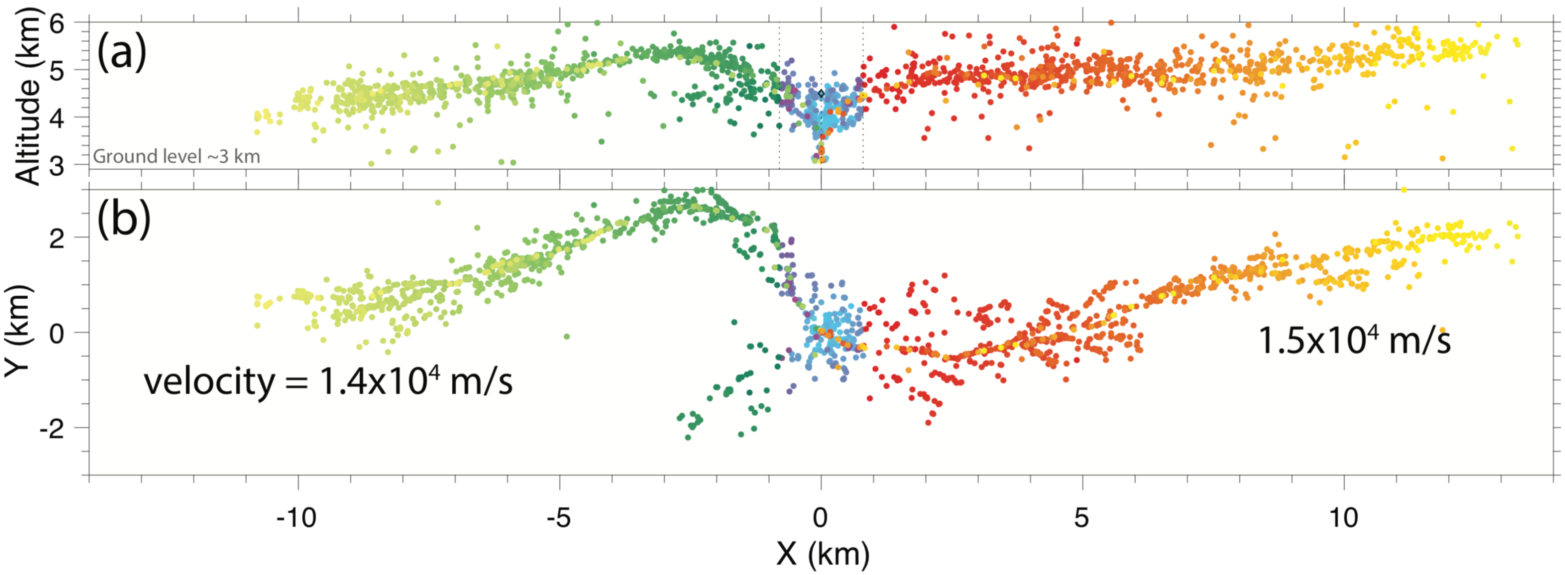
da Silva and Pasko [JGR, 2015]

Extra Slides: Rocket-Triggered Lightning

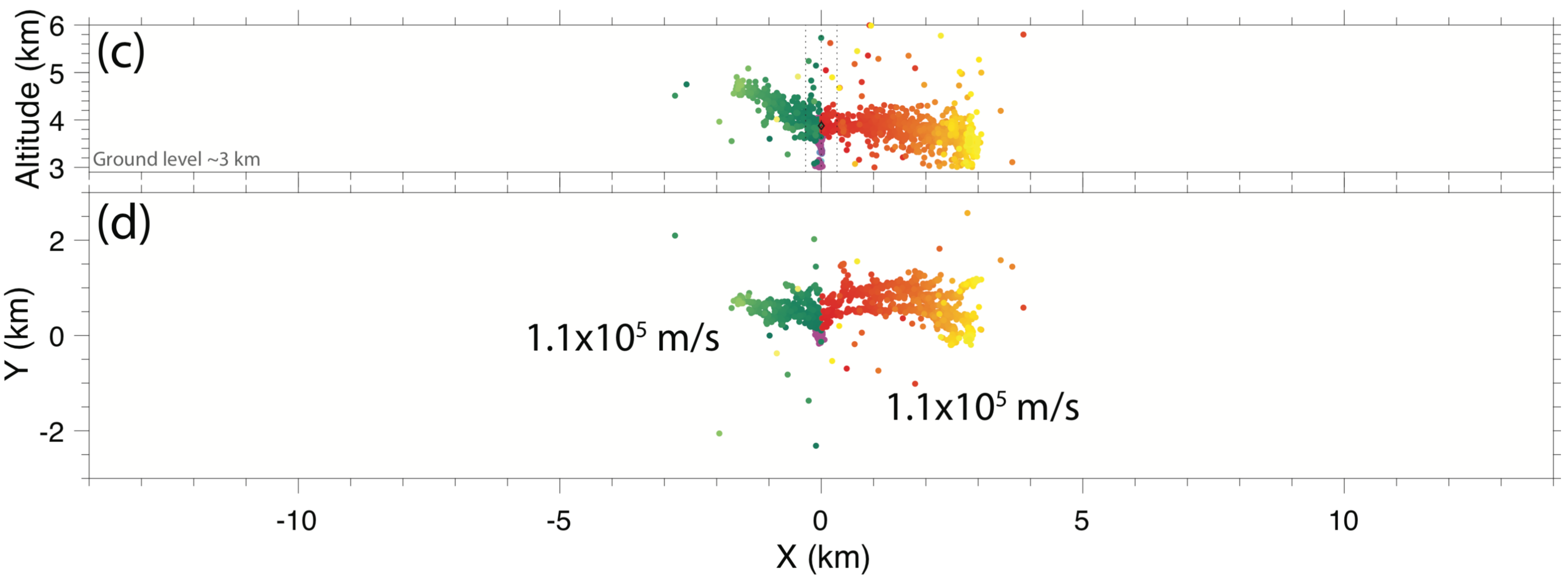
From: *da Silva et al.* [GRL, 2023]

Leaders in Triggered Lightning

(a-b) Triggered flash with upward positive leaders

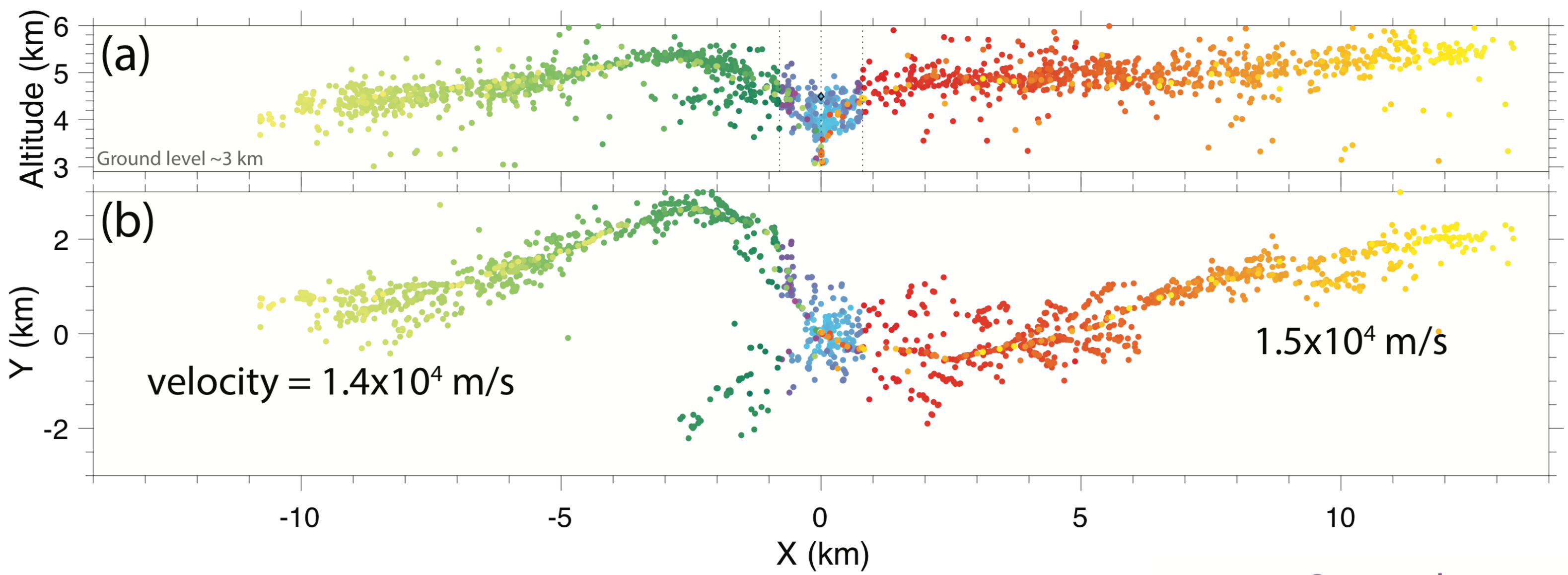


(c-e) Triggered flash with upward negative leaders

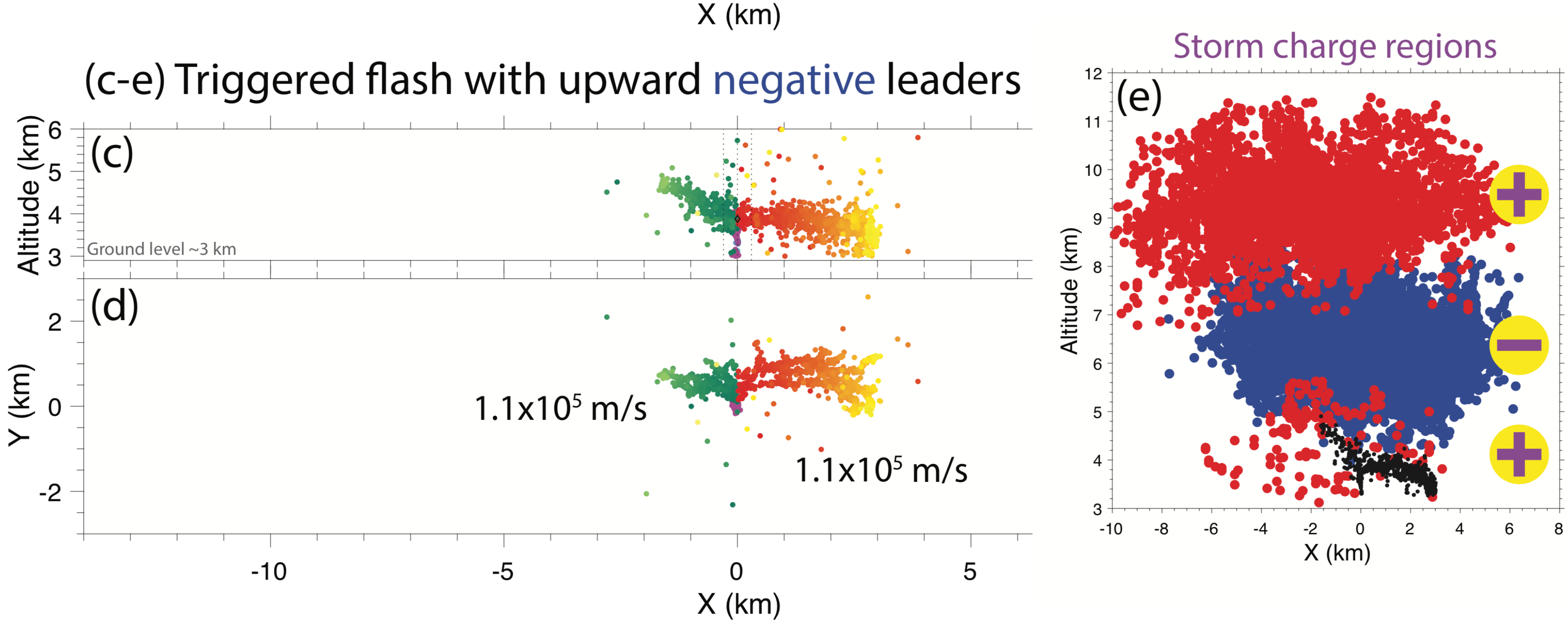


Leaders in Triggered Lightning

(a-b) Triggered flash with upward positive leaders

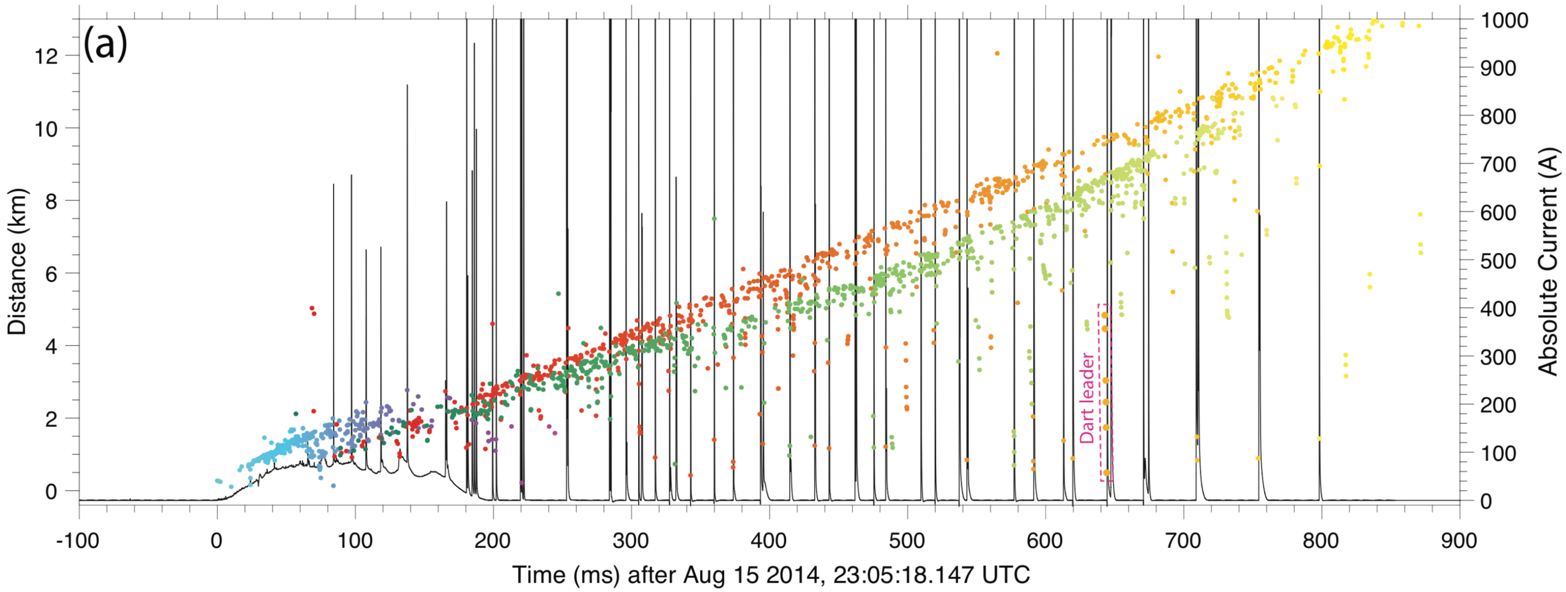


(c-e) Triggered flash with upward negative leaders

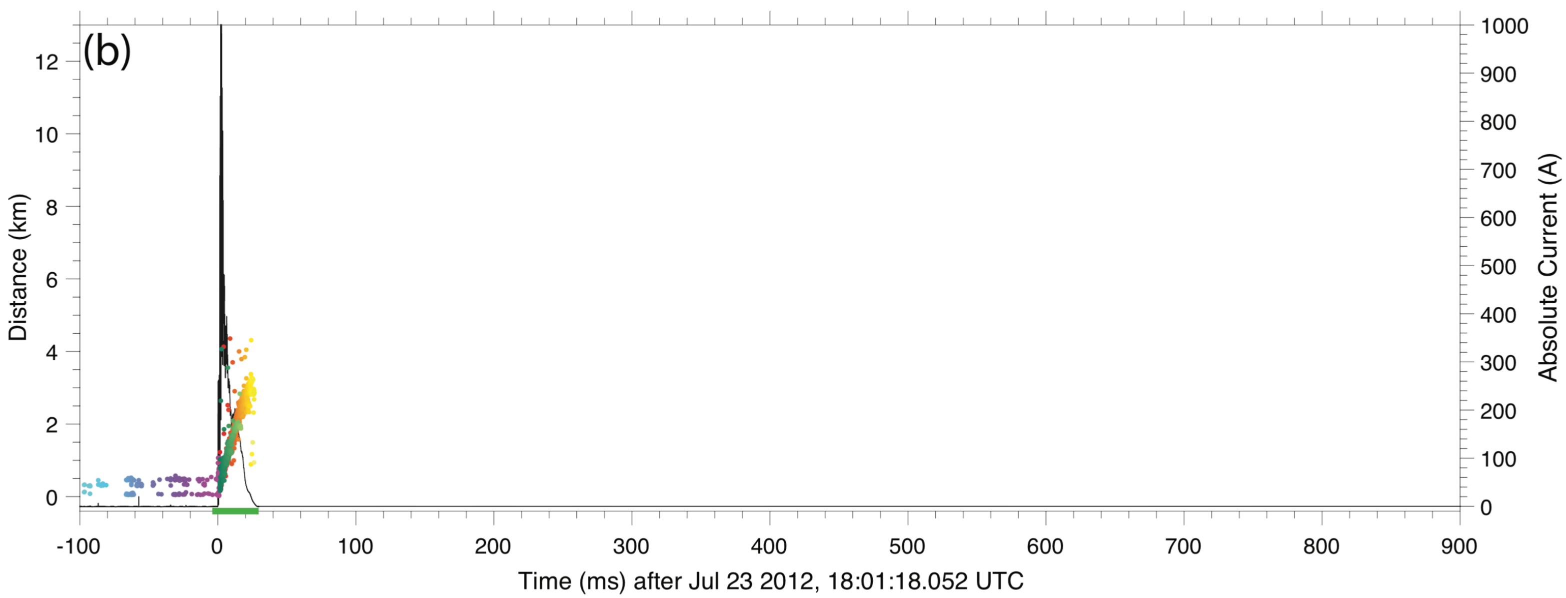


Leaders in Triggered Lightning

Triggered flash with upward positive leaders

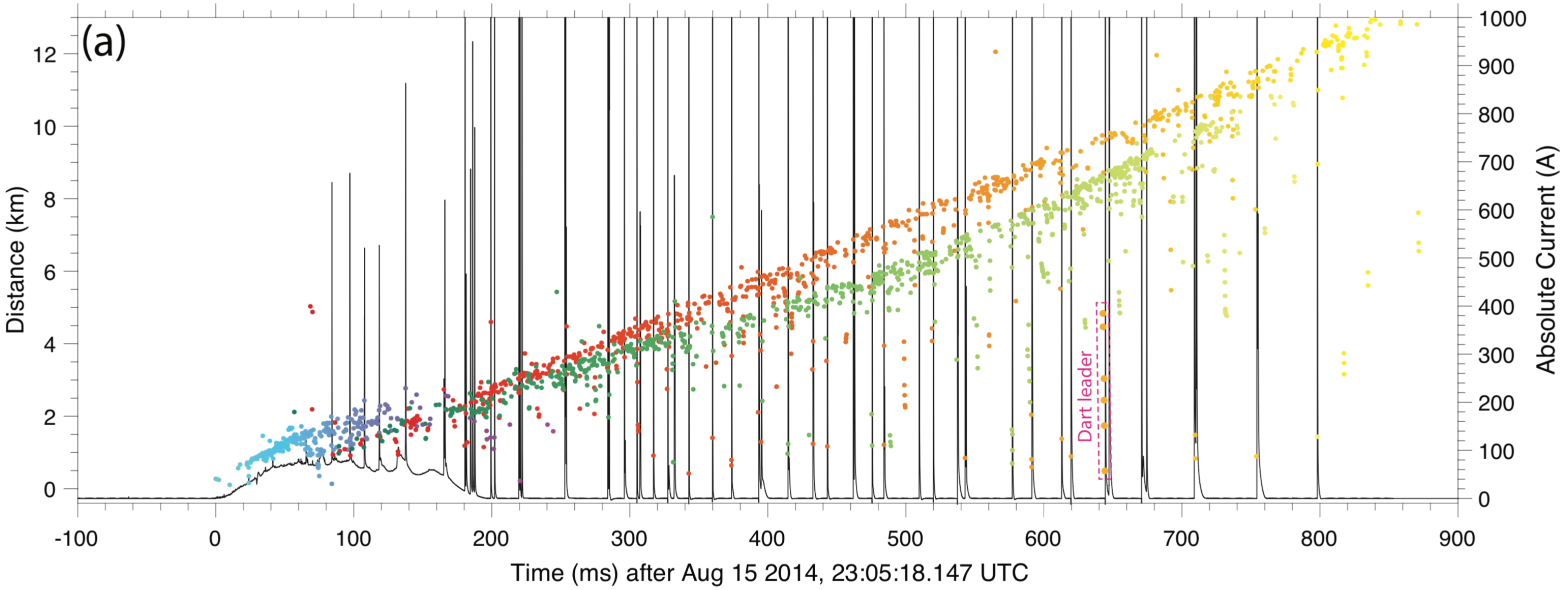


Triggered flash with upward negative leaders



Return Stroke Channel

Triggered flash with upward positive leaders



Triggered flash with upward negative leaders

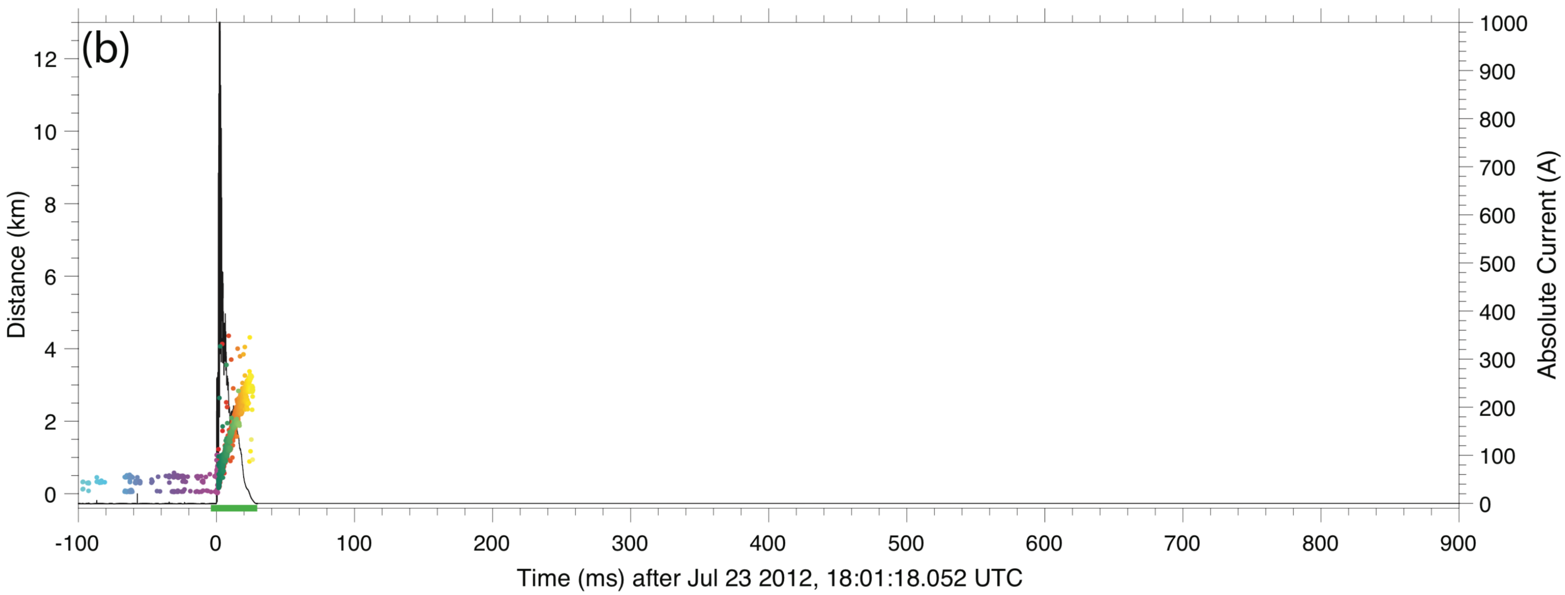


Image courtesy of H. Edens

Leaders in Triggered Lightning