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Lightning Leader Channel Modeling

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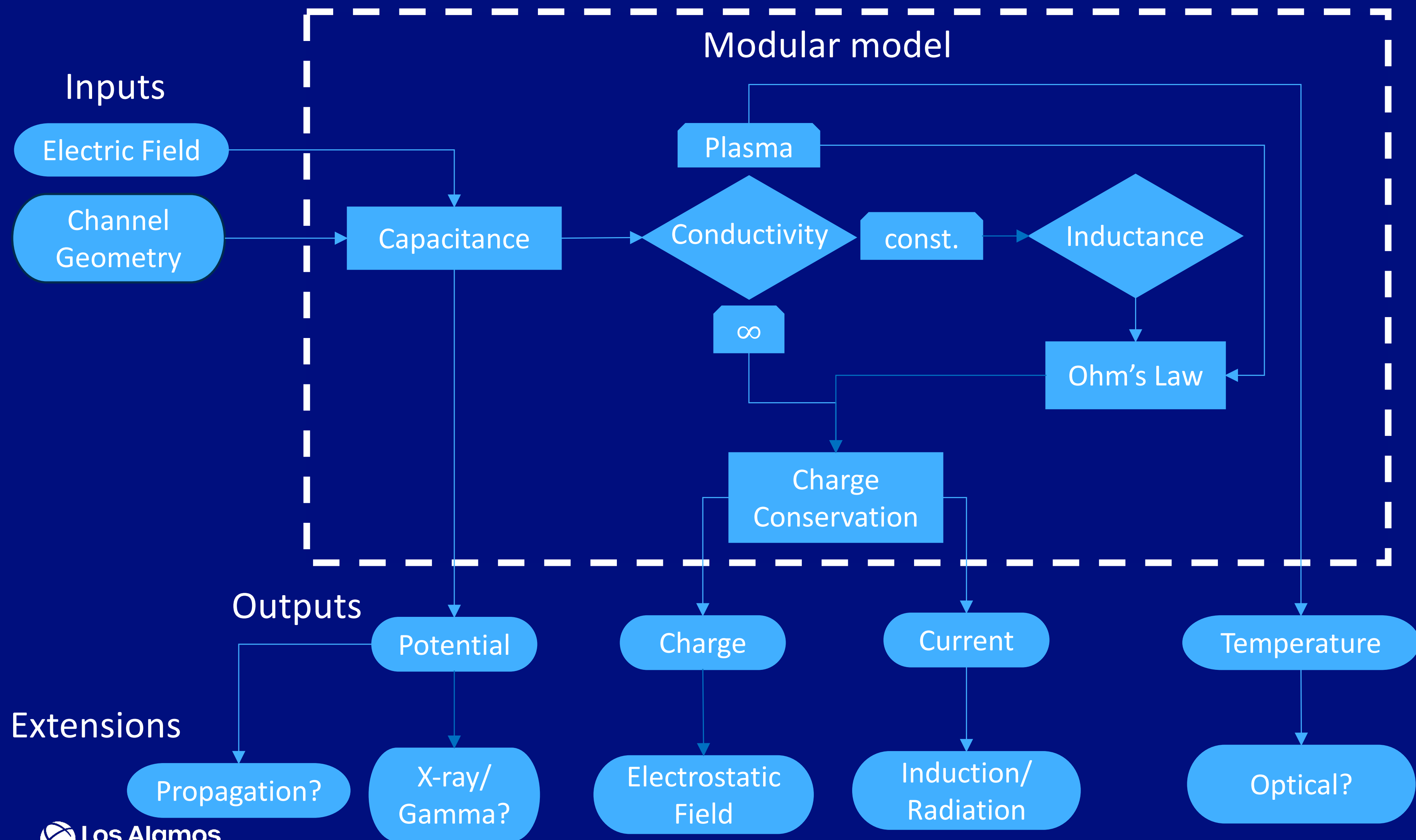
Lightning Modeling Grand Challenge Workshop, 1-3 April 2025, Texas Tech University, Lubbock, Texas


Modular model
Inputs
Electric Field
Channel Geometry
Plasma
Capacitance
Conductivity
const.
Inductance
∞
Ohm's Law
Charge Conservation
Outputs
Potential
Charge
Current
Temperature
Extensions
Propagation?
X-ray/ Gamma?
Electrostatic Field
Induction/ Radiation
Optical?

Equipotential Leader Model (Capacitance Only)

Kasemir (1960)

Mazur & Ruhnke (1993,1998)

Behnke et al. (2005) → Propagation

Celestin & Pasko (2011) → X-ray/Gamma

Pasko (2014) → Field change

Jensen et al. (2024) → Field change

Similar to fractal models, but more accurate capacitance (in 1D)

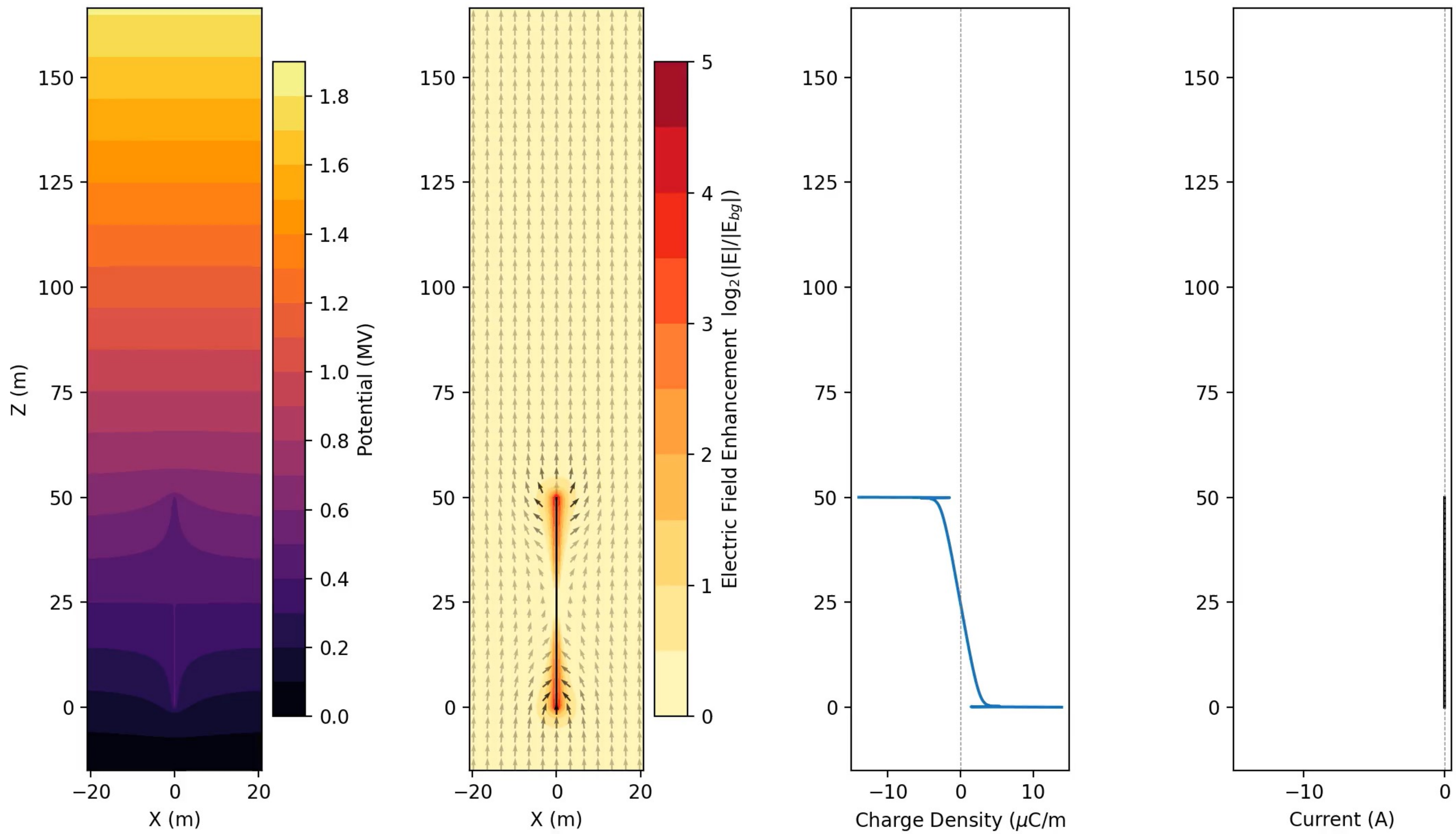
Capacitance

$$\Phi(s) = \Phi_{amb}(s) + \frac{1}{4\pi\varepsilon_0}\int_{s_a}^{s_b} \frac{\lambda(s')}{\sqrt{(s-s')^2 + r_C^2}} ds'$$

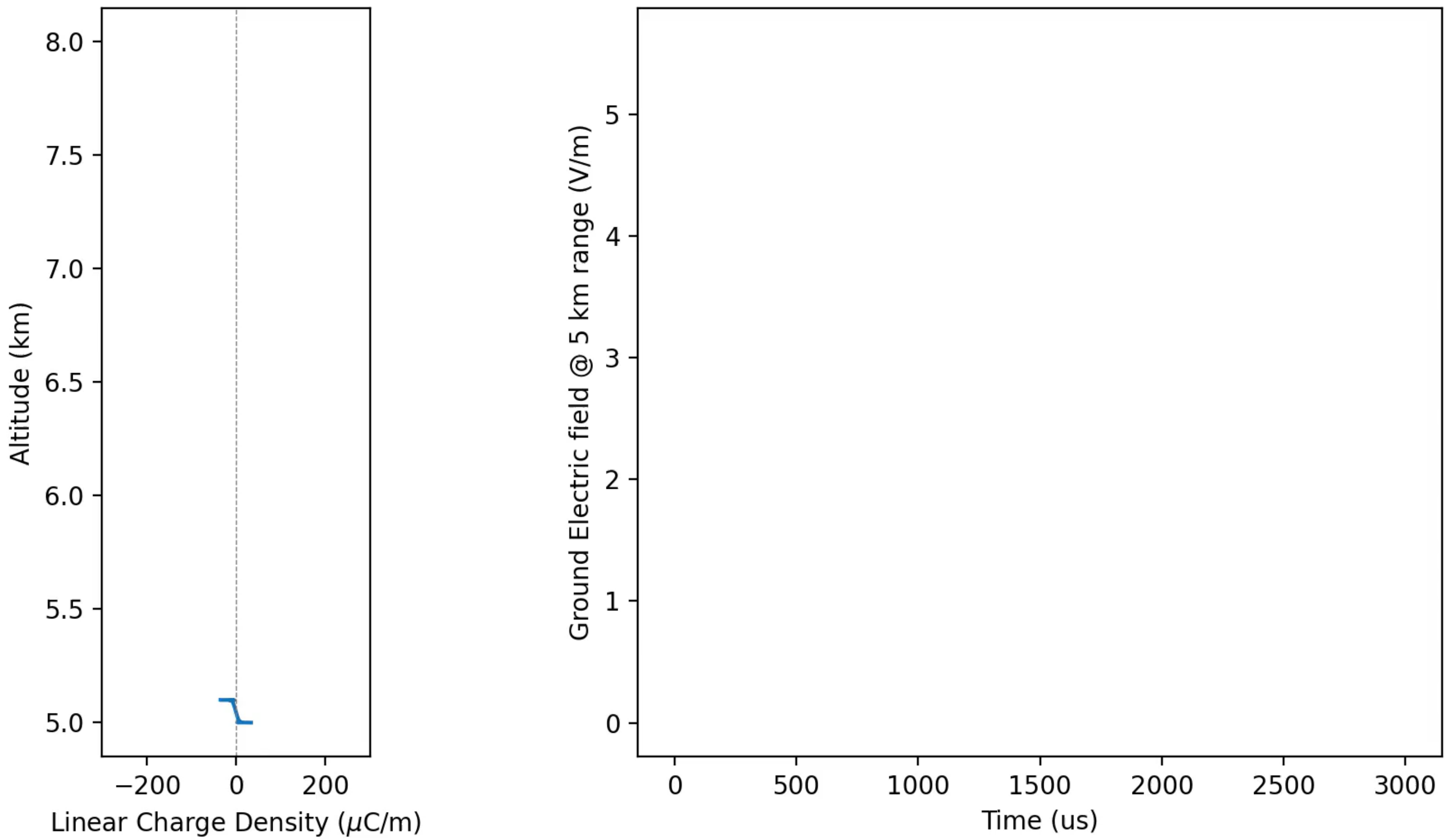
$$\Phi_{cha} = \frac{1}{s_b - s_a}\int_{s_a}^{s_b} \Phi_{amb}(s) ds$$

$$\frac{\partial \lambda}{\partial t} + \frac{\partial I}{\partial s} = 0$$

Equipotential Leader Model



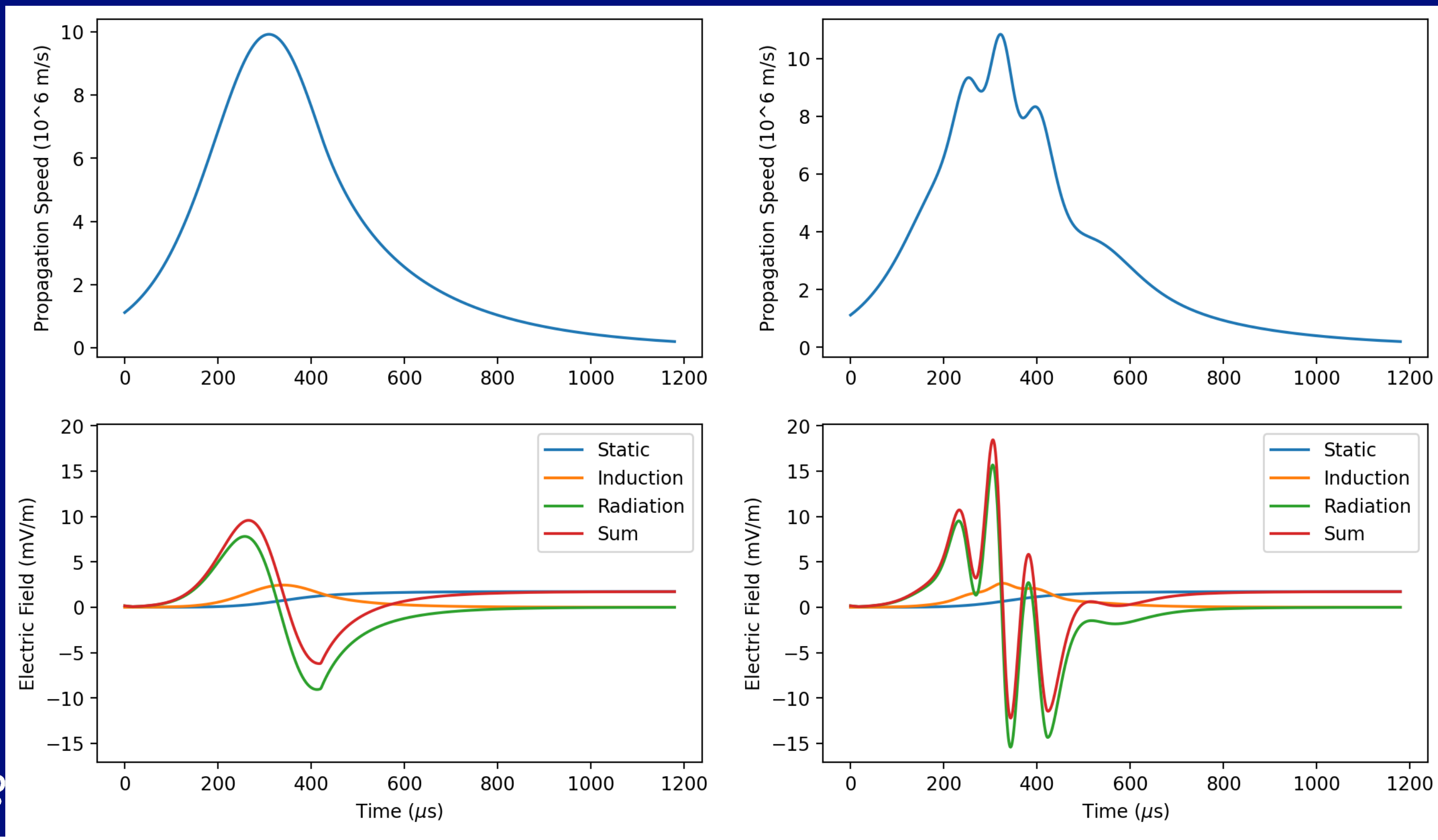
Electrostatic Field Change (e.g. Pasko, 2014)



Induction, and Radiation fields (at 100 km)

Equation 21 from Shao (2016), Uman et al. (1975)

Equipotential model radiation field is overly sensitive to $\frac{dv}{dt}$



Capacitance + Resistance

Pantuso & da Silva (2024)
→ Propagation

$$\Phi(s,t) = \Phi_{amb}(s) + \frac{1}{4\pi\varepsilon_0}\int_{s_a}^{s_b} \frac{\lambda(s',t')}{R(s,s')}\,ds'$$

Explicit time dependence
(dart leader radiation field?)

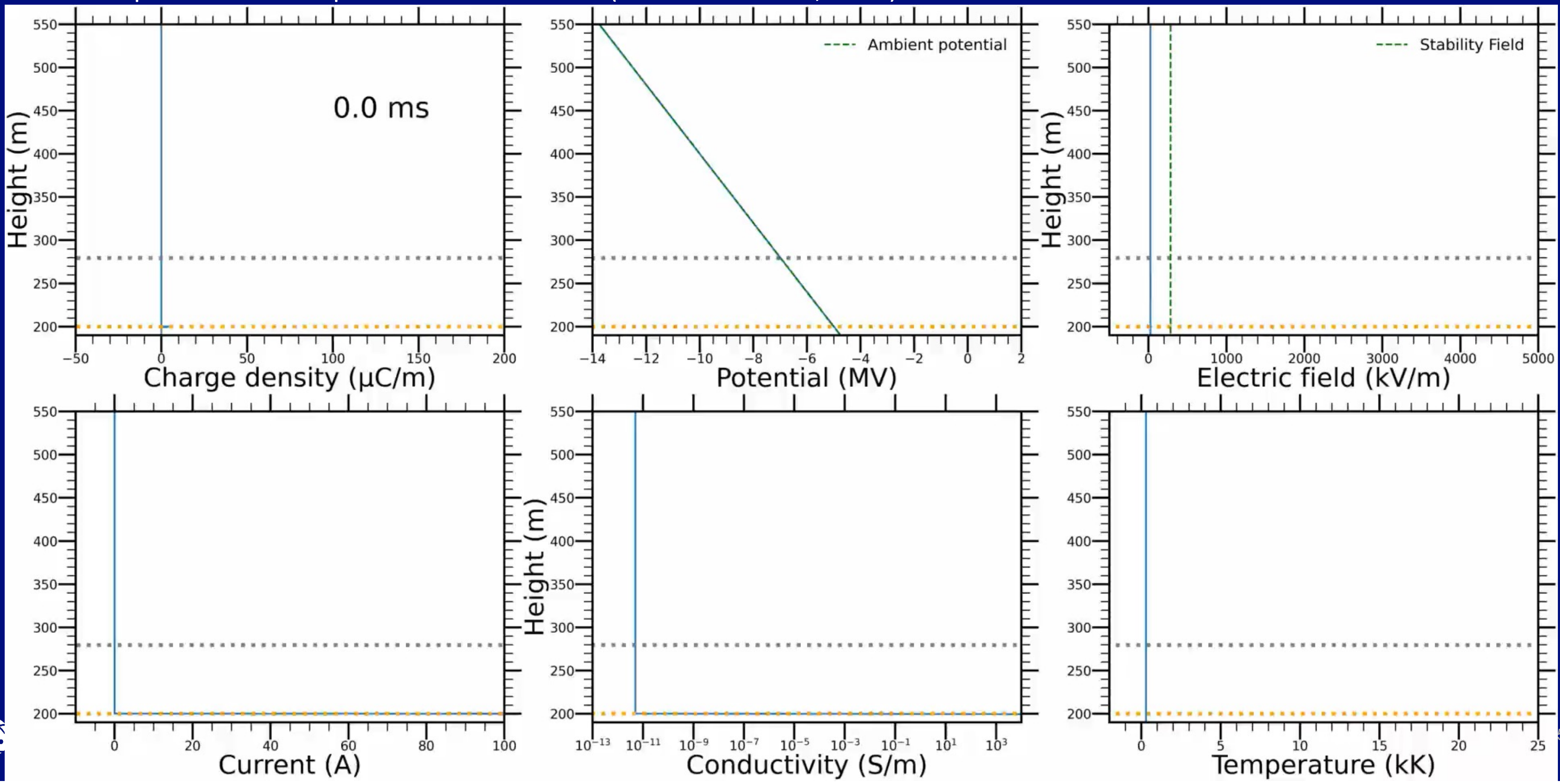
$$I(s,t) = -\sigma\pi r_R^2 \frac{\partial}{\partial s}[\Phi(s,t')]$$

Optional plasma dynamics

$$\frac{\partial\lambda}{\partial t} + \frac{\partial I}{\partial s} = 0$$

Adding Plasma Thermodynamics Model (Pantuso et al. 2024, AGU)

Computationally expensive at resolution needed for self-consistent propagation
Could add more physics, e.g. da Silva et al. (2019)
Temperature output could drive optical emissions model (Wemhoner et al., 2025)



Capacitance + Resistance + Inductance

Moini et al. (2000) → Return stroke
Carlson et al. (2010) → TGF
Lian et al. (2014) → Return stroke
da Silva & Pasko (2015) → Leader step fields

Accurate transient behavior

$$\Phi(s,t) = \Phi_{amb}(s) + \frac{1}{4\pi\varepsilon_0}\int_{s_a}^{s_b}\frac{\lambda(s',t')}{R(s,s')}ds'$$

$$A_s(s,t) = \frac{\mu}{4\pi}\int_{s_a}^{s_b}\frac{I(s',t')}{R(s,s')}ds'$$

$$I = -\sigma\pi r_R^2\left(\frac{\partial A_s}{\partial t} + \frac{\partial \Phi}{\partial s}\right)$$

$$\frac{\partial \lambda}{\partial t} + \frac{\partial I}{\partial s} = 0$$

Summary

- Capacitance only model
  - First approximation potential/charge/current for average leader extension
    - Electrostatic fields
- Capacitance + Resistance
  - Explicit time dependence → induction and radiation fields for slow processes?
  - Optional: Plasma Dynamics
    - Leader cutoff → Dart leaders
    - Propagation?
    - Temperature → Optical output?
- Capacitance + Resistance + Inductance
  - Accurate handling of transients (leader steps, return strokes, etc)
    - More accurate induction and radiation fields

Major limitations:

- All of these models are currently 1D. Need to be extended to branching 3D channels
- Several assumed parameters depend un-modeled streamer/corona processes

Constant conductivity, 1 km conducting channel in $E_{bg} = 10$ kV/m